SUFFOLK UNIVERSITY

INDUSTRIALIZED GROWTH IN DEVELOPING ECONOMIES

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DEDICATION

I humbly dedicate this dissertation to my parents Zari and Ahad who have supported me throughout my life. They are the ones who planted the seeds of the forever quest for knowing in my essence, thus the tree and its fruits all belong to them.

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PREFACE

ABSTRACT

This thesis provides a model of growth and debt accumulation for developing countries aiming to overhaul their economies through industrialization. The core of the model is an infant R&D sector that allocates labor to generate knowledge and uses the knowledge to generate machinery used across the economy. This sector is initially subsidized and trade protected. The subsidy and protection will be relieved after the R&D sector reaches a self-sufficient production level where the cost of producing machinery is less than its marginal productivity in the other sectors. Based on this model, I try to answer several key questions: first, how much subsidy should be applied to the machinery prices to maximize the output of the economy, and second what growth level is attainable in a such an economy.

INTRODUCTION

The goal of this thesis is to study the impact of industrialized growth in an open economy. Specifically, we want to analyze the impact of two major exogenous variables, government subsidization and foreign borrowing, on an economy's output and its growth rate. We start with a simple closed economy model in the first chapter, and study the agents and their interactions. This simplification allows us to define the mathematical framework used throughout the thesis. We consider three agents: the R&D sector, producers, and the government. A traditional producing economy trying to transition itself and produce non-traditional goods allocates a portion of its endowment to conduct research, and uses it to manufacture tools used in the primary sector. The model we use to study the interaction between the R&D sector, and the rest of the economy, is a generalization of Romer's innovation-based theory. While Romer considers a special equilibrium where the growth rates of supply and demand are equal, and considers a unit elasticity of production for the labor employed in the R&D sector, we define the equilibrium in our complete open economy model based on economy's stable debt-to-output ratio. We also consider a general elasticity of production for the labor employed in the R&D sector.

Due to the low marginal product of human capital in the early stages of industrialization, the government plays an imperative role by subsidizing the human capital used in production. Although subsidization has a positive impact, we see that it is limited in size and can cause income inequality. Besides the mathematical framework provided in this thesis we should consider that societal factors of this sort are very important and can, if not controlled, cause turmoil and instability defeating the original purpose of increasing growth. In fact, duality in income was one the major reasons that industrialization failed in

Latin America's import-subsidized industrialization (Frieden, 2006). In the second chapter, we thoroughly consider the impact of industrialization on wages and income per capita.

The limited possible growth in a closed economy via subsidization is one of the main reasons for the economy to open to foreign borrowing. In the final chapter, we consider an open economy, and study the impact of foreign borrowing on growth. Borrowing not only is used to fuel the growth, it also helps in fighting income inequality, and to maintain a more homogenous standard of living in the society. We present specific borrowing policies on how much should be borrowed to optimize growth and maintain a minimum level of income per capita. The last contribution of this thesis is to look at default, specifying the optimum level of debt, and analyzing some cases that can destabilize the economy.

The general framework used to shape the mathematical model presented in this thesis follows the turnpike theory which states that given the current level of growth and a finite-horizon plan for the level of growth, the planner's best approach would be to optimize just over a finite-horizon. The growth path need not be on the balanced growth path itself, but just stay in the vicinity of it over the planning period.

RELATED RESEARCH

The model I am proposing belongs to the endogenous growth class of theories that explain long-run growth as emanating from economic activities that create new technological knowledge. The first version of endogenous growth theory was AK theory, which did not make an explicit distinction between capital accumulation and technological progress (Howitt-and-Aghion, 1997). In effect, it lumped together the physical and human capital whose accumulation is studied by neoclassical theory with the intellectual capital that is accumulated when innovations occur. An early version of AK theory was produced by (Frankel, 1962) who argued that the aggregate production function can exhibit a constant or even increasing marginal product of capital. Romer (Romer P. , 1986) produced a similar analysis with a more general production structure, under the assumption that saving is generated by intertemporal utility maximization instead of the fixed saving rate of Frankel. Lucas (Lucas, 1988) also produced a similar analysis focusing on human capital rather than physical capital; following (Uzawa, 1965) he explicitly assumed that human capital and technological knowledge were one and the same.

AK theory was followed by a second wave of endogenous growth theory, generally known as innovation-based growth theory, which recognizes that intellectual capital, the source of technological progress, is distinct from physical and human capital. Physical and human capital are accumulated through saving but intellectual capital grows through innovation. One version of innovation-based theory was initiated by (Romer, 1989) who assumed that aggregate productivity is an increasing function of the degree of product variety. The other version of innovation-based growth theory is the Schumpeterian theory developed by Aghion and Howitt (Aghion, 1992),and Grossman and Helpman (Grossman, 1991).

Romer's model specifically is one of the building blocks in the models presented in this thesis. While Romer's novel approach introduced a new apparatus on how the R&D sector plays a role in production, it was too specific. First, the general equilibrium considered by Romer was a special case where the growth on the demand and supply sides of the economy were equal. While this sounds natural, it is not necessary. The general equilibrium presented in this thesis uses the stabilizing amount of debt and shows how Romer's equilibrium is only a special case of the overall equilibrium. Second, representation of the product elasticity of labor in the R&D sector is rather unrealistic. While in general we consider a varying elasticity – like β in the widely-known Cobb-Douglas model – and use econometrics to specify its true value, he considers a unit elasticity. Some of the results he obtains are directly based on this choice; for example, the level of labor employed in the R&D sector becomes constant in Romer's model. In this thesis, however, I have used a generic elasticity and show how specific values like 1 or 0.5 simplify the model and can be used to explain some of the derivations.

(Frankel, 1962), was first to introduce human capital as a primary factor in innovation-based growth. In our model, we also consider human capital to be transferring knowledge from the R&D sector into the primary sector. Unlike Frankel, who uses typical Solow steady state growth, however, the rate of change of capital intensity in our model is dynamic and we cannot make any specific assertion - just based on the information we have - on whether is it converges to a steady state or not. As we will see in chapter 3, the limiting factor for growth is the government budget available to fund the subsidies, and we will use this limit to find the steady state growth.

CHAPTER 1

SUPPLY SIDE DYNAMICS IN CLOSED DEVELOPING ECONOMIES

In this chapter, we focus on the supply side, and see how the economy can grow based on the advancement in technology. The advancement can occur alongside production under learning by doing, or could be achieved in a dedicated research and development department. In any case, in the initial stages of development, we suppose that the marginal product of the machinery produced in the R&D sector is lower than the threshold required to make this sector profitable and hence self-sufficient. As I will show, this threshold is significant in deterring private investors – even with perfect foresight - from entering the market. Therefore, the public sector needs to provide a subsidy to make this sector grow early on. The source of subsides can be internal tax revenues or external debt. In this chapter, I look at a closed economy where the government imposes taxes on firm profits and uses the proceeds to subsidize the price of machinery.

The model I present in this study is based on a three-sector economy that resembles what normally exists in countries aiming to overhaul their traditional primary-producing economies by creating a core industrial sector that could increase the growth of their tradable and non-tradable sectors.

While other scholars like (Romer, 1989) and (Frankel, 1962) have focused on the impact of the R&D sector in theory, this study considers the entire economy and answers several practical questions including: can an infant R&D sector can be self-reliant, or does it need government aid? How much subsidy must government allocate to the R&D sector to make it cost-effective for the primary sectors? And how will the economy's output change throughout the growth period?

CLOSED ECONOMY MODEL

In the status quo, the economy is highly engaged in primary tradable and non-tradable production. To increase long-term growth, an R&D sector is formed that uses part of its labor to generate knowledge, and the rest of its labor and capital to produce machinery. The machinery, alongside the knowledge transferred by the virtue of using it, boosts the entire economy. On the supply side, firms in all three sectors - R&D and primary - maximize their profit. (Table 1 describes the variables used in the chapter.

By setting the tax rate and the subsidy allocation, government has a prime role in directing production. Government intervenes for two reasons. First, in the initial stages of the

Table 1. Variable Description

Parameter	Description
$\boldsymbol{K_{i,t}}$, $(\boldsymbol{k_{i,t}})$	Capital (per worker) used in sector *i*, at period *t* where *i* can be *T* for tradable, *N* for non-tradable, and *M* for machinery. For example, $K_{T,t}$, indicates the capital used in the primary sector in year *t*.
$\boldsymbol{L_{i,t}}$	Labor employed in sector *i*, at period *t*. For example, $L_{N,t}$, indicates the labor employed in non-tradable sector at year *t*.
$\boldsymbol{M_{i,t}}$ $(\boldsymbol{m_{i,t}})$	Machinery (per worker) used in sector *i*, at period *t*. For example, $M_{M,t}$, indicates the machinery used in the machinery sector at year *t*.
$\boldsymbol{A_{i,t}}$	Technology applied in sector *i*, in period *t*.
$\boldsymbol{\Pi_i}$	Net present value of profits (or losses) in sector *i*.
$\boldsymbol{\pi_{i,t}}$	Profit of sector *i*, generated in period *t*.
$\boldsymbol{p_{M,t}}$	Price of a machine relative to a unit of tradable goods generated in period *t*.
$\boldsymbol{p_{N,t}}$	Price of a unit of non-tradable good relative to a unit of tradable good generated in period *t*.
$\boldsymbol{w_t}$	Wages in period *t*.
$\boldsymbol{r}$	Real interest rate which is assumed constant over the entire industrialization.
$\boldsymbol{\tau_t}$	Tax rate at period *t*.
$\boldsymbol{s_{i,t}}$	Subsidy given to sector *i*, at period *t* to buy machinery.
$\boldsymbol{Y_{i,t}}$	Output of sector *i*, at the end of period *t*.
$\boldsymbol{\theta}$	Learning-by-doing factor.

industrialization, the marginal product of machinery is low and the profit from producing it could be negative in the R&D sector. Second, by subsidizing the price of machinery, it creates an incentive for the R&D sector to produce more machinery, which will in turn increase growth.

Figure 1 shows the tasks performed by each agent and the interactions among them. The interaction between the individual firms and R&D sector is best described as a cooperative game where each party tries to maximize its profits knowing that its decision has impacts on the other party's behavior. Based on the amount of machinery ordered, the R&D sector hires research staff who will generate the technology and knowledge used to produce the advanced machinery to sell to the individual firms. Government uses the taxes collected in the previous period to allocate the subsidy to the firms, based on how much machinery they use in their production. Given the sheer number of firms, and therefore lack of cooperation among them we assume firms do not aim at a Nash Equilibrium with respect to Government, and so do not optimize their purchase of machinery to obtain as much subsidy as possible.

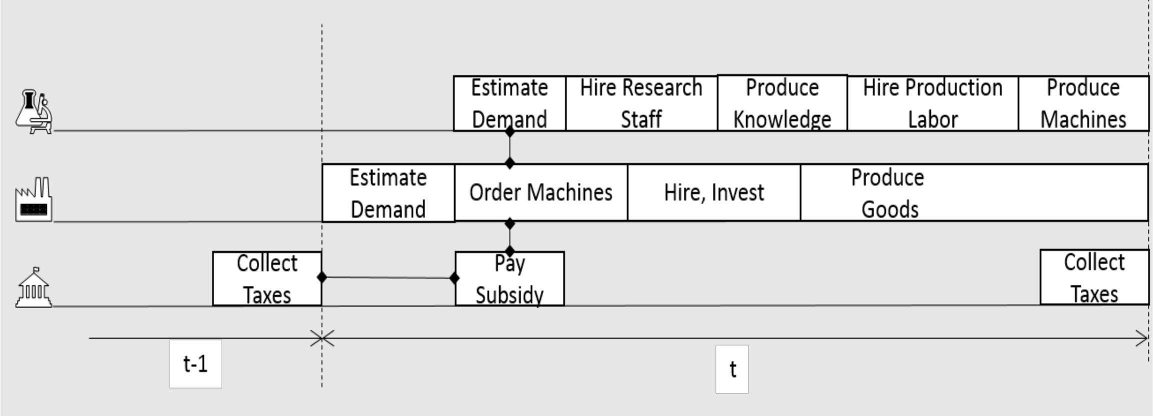

Figure 1. Activities and Decisions for agents included in the model

THE R&D SECTOR

The R&D sector is based on Romer's endogenous technological change model (Romer, 1989). Like Romer, I assume that the increase in knowledge is a function of R&D labor. In contrast to Romer's model, the R&D sector does not solely generate knowledge. It splits its labor into research and machine production. Equations (1) and (2) show the dynamics of the research and machine production in the R&D sector. A_t is the knowledge gained in the R&D sector and $A_{M,t} = h(A_t)$ is the productivity in producing machinery as a function of knowledge gained in the R&D sector where we should have $A_{M,t} \leq A_t$. $L_{A,t}$ and $L_{M,t}$ are the labor employed in the R&D and primary sectors accordingly, and M_t is the amount of machinery produced.

(1) $A_{t+1} - A_t = \theta L_{A,t} A_t$

(2) $M_t = A_{M,t} H(L_{M,t}, K_{M,t})$

Not only does the R&D sector maximize its profits with respect to its production factors, which include labor allocated to research and machine production, and capital, it does it in a way to be in a Nash Equilibrium state anticipating other sectors' choice of the amount of machinery they will buy. This choice depends on the price of the machinery and the productivity these sectors gain by utilizing the machinery. To incorporate this into the model, we include the first order conditions maximizing the primary sectors' profit with respect to machinery into the R&D sector constraints – we have summed up these equations knowing that the output within each sector does not depend on the amount of machinery used in the other sector. Being protected under intellectual property rights for its innovations, the R&D sector is a price setter. Using capital as the numeraire, the R&D

sector's profit is given by equation (3), the Lagrangian by equation (4) – where the second term comes from the market clearing condition for machinery setting its supply equal to the demand from the primary sector: by subtracting equations (22) from (23) we get the second term in the Lagrangian - and the corresponding first order conditions are given by equation (5) to (7). These equations are simultaneously solved with the primary sector's first order conditions to obtain the Nash Equilibrium – note that I have defined $T_t \stackrel{\text{def}}{=} \frac{(1+r)(1-\tau_{t-1})}{1-\tau_t}$ and w_t is the wage.

$$\text{(3)} \quad \Pi_R = \sum_{t=0}^{\infty}(1-\tau_t)\left(\frac{1}{1+r}\right)^t \left[p_{M,t}M_t - w_t\left(L_{A,t} + L_{M,t}\right) - \Delta K_{M,t}\right]$$

$$\text{(4)} \quad \begin{aligned} \Lambda_R = \sum_{t=0}^{\infty}(1-\tau_t)\left(\frac{1}{1+r}\right)^t &\left[p_{M,t}M_t - w_t\left(L_{A,t} + L_{M,t}\right) - \Delta K_{M,t}\right] \\ &- \lambda_{y,t}\left[\frac{1}{2}\left(\frac{\partial y_t}{\partial M_t}\right) + p_{M,t} - s_t - T_t(p_{M,t-1} - s_{t-1})\right] \\ &- \lambda_{M,t}\left(M_t - A_{M,t}H(L_{M,t}, K_{M,t})\right) \end{aligned}$$

$$\text{(5)} \quad K_{M,t}\colon \lambda_{M,t}h(A_t)H_K(.) + 1 = T_t$$

$$\text{(6)} \quad L_{M,t}\colon \lambda_{M,t}h(A_t)H_L(.) = w_t$$

$$\text{(7)} \quad L_{A,t}\colon \lambda_{M,t}\dot{h}(A_t)H(.)\frac{t\theta A_0 A_t}{A_1} = w_t$$

$$\text{(8)} \quad M_t\colon \lambda_{M,t} = p_{M,t} - \frac{\lambda_{y,t}}{2}\left(\frac{\partial^2 y_t}{\partial {M_t}^2}\right)$$

$$\text{(9)} \quad p_{M,t}\colon M_t = \lambda_{M,t} - \lambda_{y,t+1}$$

From equations (5) to (9) we can solve the for five unknowns ($K_{M,t}, L_{M,t}, \lambda_{M,t}, \lambda_{y,t}, and\ w_t$) with respect to the labor and input factors of the primary sectors – the input factors of the primary sectors will be expressed in terms of the labor hired in those sectors in the next section. Given that wages are identical for all labor, w_t, is specified in the primary sector due to its much larger size and in the R&D sector it is treated as an exogenous parameter.

Using equations (6) and (7) we obtain the labor employed in producing knowledge in terms of labor and capital used in producing the machinery:

$$(10) \quad \frac{\dot{h}(A_t)}{h(A_t)} = \frac{A_1 H_L(.)}{t\theta A_0 A_t}$$

Using equations (5), (6), and (9), we obtain equations (11) and (12). These two equations and equation (10) can jointly specify the dynamics of labor and capital in the R&D sector. Equations (10) and (11) specify the levels of factors used in the primary sector. Equation (11) shows how primary sector's productivity responds to changes in different level of production factors. Note that T_t has the same change direction as τ_t (if $\tau_t > \tau_{t-1} > \tau_{t-2} \Rightarrow T_t > T_{t-1}$ and vice versa), therefore if the government decreases the tax rate over time, a primary firm would have a higher production by having a higher labor to capital ratio. If the government keeps the tax rate the same, then a primary firm will keep the its employed labor to capital ratio the same. Equation (12) shows that the level of machinery produced by primary firms increases when the marginal productivity of labor increases or wage decreases

$$(11) \quad \frac{H_K(.)}{H_L(.)} = \frac{T_t - 1}{w_t}$$

$$(12) \quad M_t = \frac{w_t}{h(A_t)H_{L,t}(.)} - \frac{w_{t+1}}{h(A_{t+1})H_{L,t+1}(.)}$$

In Chapter 2 we will derive the explicit formula for the special case of Cobb-Douglas production functions.

PRIMARY SECTORS

Equations (13) and (14) show how technology is diffused to primary sectors – tradable (T) and non-tradable (N) - depending on the amount of machinery used in production in each sector.

$$(13) \quad A_{T,t} = A_T + \Omega\left(\frac{M_{T,t}}{M_t}\right) A_t$$

$$(14) \quad A_{N,t} = A_N + \Omega\left(\frac{M_{N,t}}{M_t}\right) A_t$$

Here, $\Omega(.)$ specifies the technology diffusion and has the following characteristics:

1. It is an increasing non-concave function.
2. Its value is zero for zero investment in machinery
3. It approaches one for significant investment in machinery, i.e., when either sector's machinery usage gets closer to the entire machinery produced.

Equations (15) and (16) specify the profit made in each sector in terms of the price of the tradable goods. Equation (17) is the government budget constraint, and states that in a closed economy the sum of taxes collected from primary and R&D sectors equals the machinery subsidy

$$(15) \quad \Pi_T = \sum_{t=0}^{\infty} (1-\tau_t)\left(\frac{1}{1+r}\right)^t \left[A_{T,t}F\left(K_{T,t}, M_{T,t}, L_{T,t}\right) - w_t L_{T,t} - \Delta K_{T,t} - (p_{M,t} - s_t)\Delta M_{T,t}\right]$$

$$\Pi_N = \sum_{t=0}^{\infty}(1-\tau_t)\left(\frac{1}{1+r}\right)^t \big[p_{N,t}A_{N,t}G\big(K_{N,t}, M_{N,t}, L_{N,t}\big) - w_t L_{N,t} - \Delta K_{N,t} - (p_{M,t} - s_t)\Delta M_{N,t}\big] \quad (16)$$

$$s_{t+1}(\Delta M_{T,t+1} + \Delta M_{N,t+1}) = \tau_t(\pi_{T,t} + \pi_{N,t} + \pi_{R,t}) \quad (17)$$

The optimum subsidy and tax levels are specified by the government. Each firm, including the R&D sector, maximizes its profits leading to the first-order conditions given by equations (18) to (23). Note that, despite knowing that government will adjust the tax and subsidy rate based on their production, firms assume tax and subsidies are fixed. This is because there are many firms in the economy and without an existing coordinating organization to set the production plan for all, each firm plans independently and cannot aim to reach a Nash Equilibrium. (If such an organization existed, taxes and subsidies would be functions of inputs, and we would obtain different first order conditions). An alternative treatment of taxation is a fixed rate for the period of industrialization, which I will use in Chapter 2. To derive these equations, we have taken derivatives of the equations (15) to (17) with respect to parameters that firms control namely the level of factors used in their production including capital, labor, and machinery.

$$K_{T,t}\colon A_{T,t}F_K(.) + 1 = T_t \quad (18)$$

$$K_{N,t}\colon p_{N,t}A_{N,t}G_K(.) + 1 = T_t \quad (19)$$

$$L_{T,t}\colon A_{T,t}F_L(.) = w_t \quad (20)$$

$$L_{N,t}\colon p_{N,t}A_{N,t}G_L(.) = w_t \quad (21)$$

$$M_{T,t}\colon \left(\frac{\partial y_{T,t}}{\partial M_t}\right) + p_{M,t} - s_t = T_t(p_{M,t-1} - s_{t-1}) \quad (22)$$

$$(23) \qquad M_{N,t}: p_{N,t}\left(\frac{\partial y_{N,t}}{\partial M_t}\right) + p_{M,t} - s_t = T_t(p_{M,t-1} - s_{t-1})$$

From equations (18) to (23) and (9) from the R&D sector, we can solve for the seven unknowns ($K_{T,t}, K_{N,t}, w_t, p_{N,t}, M_{T,t}, M_{N,t}, and\ p_{M,t}$) with respect to the labor hired in the primary sector.

THE GOVERNMENT

In a closed economy, government uses the taxes collected to subsidize the price of machinery in the initial stages of industrialization when the marginal product of machinery in the primary sector is lower than its cost of production in the R&D sector. In doing so, government's goal is to maximize the present values of final output – with respect to the subsidy rate, s_t - in the entire economy given that each firm maximizes its profits as given by equation (24) – this contrasts with the maximization problem for the firms, where they do not consider government's tax plan to set their production points.

This is because government acts based on the known economy's output, and no coordination is involved, and I assumed there is no coordination among the firms.

$$(24) \qquad \max_{\underbrace{s_t}_{s.t\ firms\ profits\ are\ maximized}} \quad \mathrm{Y} = \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t (Y_T + p_{N,t}Y_N)$$

Using a first order Taylor expansion of the form $f(K, L, M) = f_K K + f_L L + f_M M$ for Y_T and Y_N and substituting for partials based on equations (16) to (21), we obtain equation (25) stating the government maximization problem in terms of the difference between price and subsidy ($q_t \stackrel{\text{def}}{=} p_{M,t} - s_t$).

$$(25) \quad Y = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^{t} [rK + w_t L + (T_t q_{t+1} - q_t) M_t]$$

Maximizing equation (25), with respect to q_t, we obtain equation (26) specifying how the government reacts to the changes in the R&D output by setting the tax rate and using it to subsidize the price of machinery.

$$(26) \quad \frac{1-\tau_t}{1-\tau_{t+1}} = \frac{M_t}{M_{t+1}} \xrightarrow{yields} \Delta\tau_t = \frac{\Delta M_t}{M_t}(\tau_t - 1)$$

Figure (2) plots the tax rate government uses when the machinery stock grows 5% a year. As the R&D sector expands, the marginal productivity of machinery increases, and firms will be eager to invest in machinery without relying on the government subsidy.

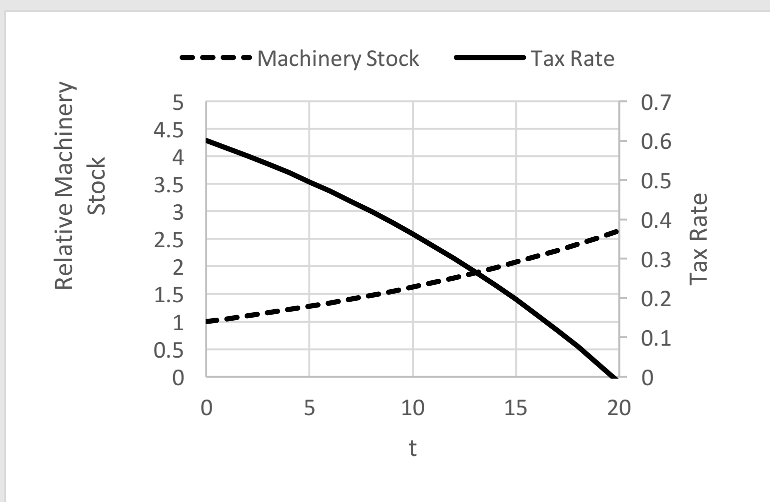

Figure 2. Government gradually decreases the tax rate as the marginal productivity of the machinery sector increases, making its marginal revenue more than its cost.

SIMULATION RESULTS

In this section, we apply the results of the previous sections to explicit forms for the production functions and study the impact of the government subsidy to the overall growth of the economy. To simplify the mathematical derivations, we assume there is only one type of primary sector, the tradable. Note that in this special case all the machinery produced in the R&D sector is consumed by the primary sector, i.e., $M_{T,t} = M_t$. For the R&D sector we assume a linear productivity factor and a linear technology diffusion function satisfying the three conditions we assumed. The specific mathematical forms are given in Table 2.

Table 2. Simulation Assumptions

Function	Special Case Used for Simulation
R&D productivity in producing machinery	$h(A_t) = A_t$
Production function for machinery	$H(L_{M,t}, K_{M,t}) = {K_{M,t}}^{\alpha} {L_{M,t}}^{1-\alpha}$
Technology diffusion function	$\Omega\left(\frac{M_{T,t}}{M_t}\right) = \frac{M_{T,t}}{M_t}$
Production function for primary sectors	$F(L_{T,t}, K_{T,t}, M_{T,t}) = {K_{T,t}}^{\alpha} {M_{T,t}}^{\beta} {L_{T,t}}^{1-\alpha-\beta}$

We have chosen a linear productivity function for the R&D machinery production and for the technology diffusion functions to simplify running the simulation; Another possible choice would be a concave function where marginal productivity would decrease when more knowledge was employed. The production functions are typical Cobb-Douglas functions of the factors used in each sector.

Using equations (10) to (12) we obtain the following expression for factors used in the R&D sector, where we have defined: $B_t \stackrel{\text{def}}{=} \frac{\alpha}{(1-\alpha)(T_t-1)}$

(27) $L_{A,t} = \frac{1}{\theta}\left[\frac{1}{1-\alpha}(\mathrm{B}_t w_t)^{\alpha} - 1\right]$

(28) $K_{M,t} = \frac{M_t}{\mathrm{A}_t}(\mathrm{B}_t w_t)^{1-\alpha}$

(29) $L_{M,t} = \frac{M_t}{\mathrm{A}_t}(\mathrm{B}_t w_t)^{-\alpha}$

(30) $(1-\alpha)M_t = \frac{{w_t}^{1-\alpha}}{\mathrm{A}_t \mathrm{B}_t} - \frac{{w_{t+1}}^{1-\alpha}}{\mathrm{A}_{t+1}\mathrm{B}_{t+1}}$

Using Equations 18 to 23 we obtain the following equations for the factors used in the primary sector, where we have defined: $\mathrm{C}_t \stackrel{\text{def}}{=} \frac{\alpha}{(1-\alpha-\beta)\,(T_t-1)}$

(31) $w_t = \left[(\mathrm{A}_T + \mathrm{A}_t)\mathrm{C}_t\left(\frac{M_t}{L_{T,t}}\right)^{\beta}\right]^{\frac{1}{1-\alpha}}$

(32) $\frac{K_{T,t}}{L_{T,t}} = \mathrm{C}_t w_t$

Finally combining equations (30) and (32) we obtain the following dynamic expressions for the optimal machinery produced in the economy we can be used with equations (22) and (26) to specify the dynamics for the economy.

(33) $(1-\alpha)M_t = \frac{(\mathrm{A}_T + \mathrm{A}_t)\mathrm{C}_t\left(\frac{M_t}{L_{T,t}}\right)^{\beta}}{\mathrm{A}_t\mathrm{B}_t} - \frac{(\mathrm{A}_T + \mathrm{A}_{t+1})\mathrm{C}_{t+1}\left(\frac{M_{t+1}}{L_{T,t+1}}\right)^{\beta}}{\mathrm{A}_{t+1}\mathrm{B}_{t+1}}$

The following values are used to run the simulations: α=0.3, β=0.35, A_0=1, θ=0.01, L=1000, r=0.05, $\mathrm{M}_0 = 1$. In these simulations, we assume labor in the primary sector is fixed whereas the R&D sector adjusts its labor - assuming that labor is freely available - in each period based on equations (27) and (29).

Figure 3 shows the result of the simulation. In this graph, we have plotted the production of the primary sector over 15 years. At years zero government introduces a subsidization plan to promote inclusion of innovations in the production. We observe that in the absence of the demand side, production takes an oscillatory path where in one period the production factors are allocated to producing machinery, dropping the primary production level, and in the next period the production resources are allocated to produce primary products creating a peak in primary production. This observation is expected in models that are based on turnpike theorem which states that in a long-term global optimization model, we might observe local deviations from the optimal path (McKenzie L. W., 1986). We see that before the economy reaches a steady level of growth – which in this simulation occurs in period 12 – every other period, a significant investment is made to the R&D sector which boosts the production for the next year. Another observation is that, without relying on external debt, it takes a rather long time – 12 years in this simulation – for the economy to reach a steady growth. The low level of output in early years of growth can cause income inequality and in extreme cases may cause societal resistance to the government growth reform. In Chapter 3, we will discuss polices that consider a balance between inequality and growth rate.

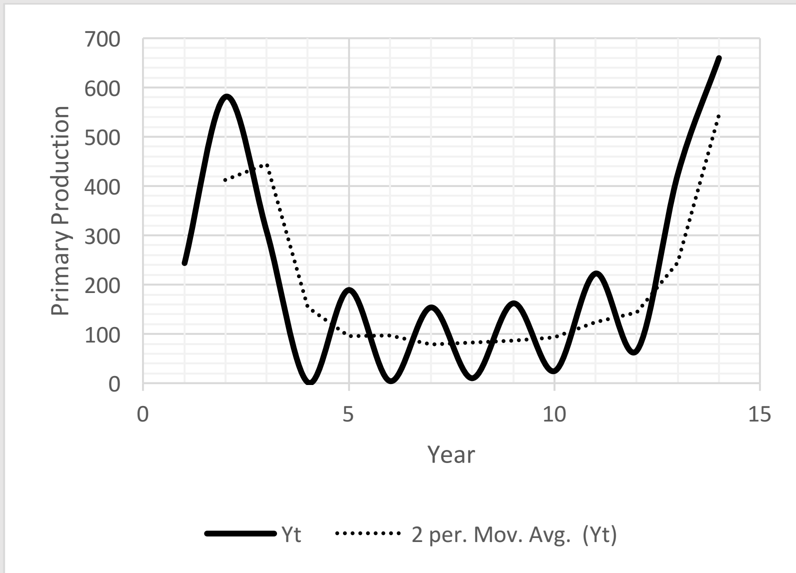

Figure 3. Economy's production. We observe an oscillatory response in absence of consumption.

In this chapter, we introduced the simplest form of our model that only includes the supply side. Using this model, we explained the interactions of three major components in the economy and showed that the trajectory the economy takes to reach its stable level of growth is oscillatory. In the next chapter, we expand the model by including the demand side which enables us to derive the closed-economy general equilibrium. We will also analyze the effect of different subsidy rates on the macroeconomic parameters including output, income-per-capita, and wages.

CHAPTER 2

GENERAL EQUILIBRIUM GROWTH IN CLOSED ECONOMIES

In chapter 1, we saw a DSGE model for the supply side including three sectors: primary sector, R&D sector, and the government, and used it to analyze the growth path for a closed economy on the path to industrialization.

In this chapter, we look at the general equilibrium for this economy by including the demand side. We start by elaborating on the R&D sector model and then specify the growth rates for both the supply and demand sides. We continue by analyzing the general equilibrium for the closed economy. We conclude by presenting simulation results that show how different levels of subsidization impact macroeconomic conditions.

The differentiating factor between Latin America's failed import-substitution-industrialization endeavors and East Asia's successful export-oriented-industrialization endeavors (Dijck, 1987) is the market for their produced goods. In the former case, production was aimed inward with little competition and lower challenges for high-quality products; in the latter case, production was aimed outward with major international competition and demand for higher-quality goods. To incorporate this effect, I include the demand side of the economy in this chapter. The demand side drives the amount produced in the primary sectors, which will in turn determine the demand for the R&D sector. The greater the demand for machinery, the higher its production and the faster the rate of knowledge generation and growth of the economy.

To continue building our model on top of the model presented in the previous chapter we apply two modifications which are listed below:

1. Following Frankel (Frankel, 1962), we have expanded the primary production function by including both physical capital and human capital. In the previous

chapter, the R&D sector produced machinery, which entered the production function as an identical factor to capital and labor. Here we assume there are different kinds of human capital that a producer can choose to employ in production, if doing so is profitable. This assumption improves the way the R&D sector's output, knowledge, is integrated into the firms' production.

2. We have added an elasticity of labor productivity to the R&D sector. In the original Romer model this elasticity is set to one, and even though the population is growing, it results in a constant level of labor being hired in the primary sector. However, the amount of labor hired in the primary sector changes as the population and marginal productivity of labor in different periods change.

I have made several assumptions about firms and government. These assumptions are listed below:

1. The firm's decision-making horizon is short compared to the government's. Therefore, in our derivations we maximize firm profits for each period separately. Previously we formed our model by considering all the periods together and assuming the producer would optimize with such a long-term goal in mind. However, a more realistic model – specifically when management is trying to maximize the value of its stock – is to assume a firm does the optimization for shorter periods. Note that given the long horizon of government decision-making that can potentially spam decades, firm's plan is considered rather short which justifies the assumption.

2. In contrast to chapter 1, I assume that the tax rate remains fixed for the duration of industrialization. The justification for this assumption is that governments typically plans ahead of launching the industrialization program which includes setting the tax rate. Normally the tax rate stays at the planned level for a longer time than the duration of the industrialized growth.
3. We assume that the ratio of direct taxation collected to the total tax revenue remains the same over the period of industrialization. While, this assumption lets us simplify the derivation of the government's limit to subsidize the R&D sector in a closed economy, one could take a different approach and use a dynamic pattern of taxation throughout the industrialization period.
4. We assume that the government subsidizes the use of human capital. Alternatively we could have assumed that the government subsidizes the R&D sector by paying a lump-sum transfer to this sector. Although the formulation could have been different, both approaches would result in the same reduction in the human factor prices.

MODIFICATIONS TO THE MODEL

The structure of the model stays intact compared to what we saw in the first Chapter. We have an R&D sector that produces knowledge. Once the knowledge is generated it is licensed to specific intermediate firms – consider this a monopoly in the R&D sector in selling the knowledge they produce. The intermediate firms in turn generate human capital using this knowledge within the same period under perfect competition. Finally, human capital will be employed in the primary sector in the following period. Government subsidizes the investment in the human capital to make it profitable for the primary sector to use it in its production specifically in the early stages of the industrialization.

For the R&D sector we use the knowledge-driven specification suggested by Romer. In contrast to the original Romer's model - which uses a linear growth function for knowledge - we use an elasticity lower than one. This is because there is no reason to assume that the labor hired in the primary sector has an elasticity of production of $\beta < 1$ whereas the labor in the R&D sector is more efficient and has an elasticity of 1. In Equation 33, θ is the knowledge diffusion factor indicating how much the knowledge generated in the current period accelerates knowledge generation in the next period.

$$(33) \quad A_{t+1} - A_t = \theta {L_{A,t}}^{\varphi} A_t$$

The production function for firms has been changed from equation (15) - where $Y = A_{T,t} F\left(K_{T,t}, M_{T,t}, L_{T,t}\right)$ - to equation (34).

$$(34) \quad Y = {L_{Y,t}}^{\beta} {K_{Y,t}}^{\alpha} \sum_{i=0}^{A_t} {h_{i,t}}^{\gamma}$$

Based on this function three factors play a role in production: labor (L_Y), Capital (K_Y), and human capital (h_i). A_t denotes the number of types of human capital – specified by the index i - which are invented in the R&D sector and produced by the intermediate firm in the same period. Firms can choose to employ different amounts of each type in their production. Using this production function, we can rewrite the profit equation as Equation 35.

$$(35) \quad \Pi_{Y,t} = (1 - \tau_t) \left[p_{y,t} {L_{Y,t}}^{\beta} {K_{Y,t}}^{\alpha} \sum_{i=0}^{A_t} {h_{i,t}}^{\gamma} - w_t L_{Y,t} - r_t K_{Y,t} - \sum_{i=0}^{A_t} p_{i,t} h_{i,t} + \sum_{i=0}^{A_t} s_t p_{i,t} h_{i,t} \right]$$

In equation (35), τ_t is the tax rate and $p_{i,t}$ and $s_{i,t}$ are the price and subsidy on human capital of type i. Tax and subsidy rates are between 0 to 1.

Before maximizing profit, we find the conditions under which a firm will decide to employ human capital either with or without government intervention.

Proposition 1. Without government intervention, a firm will employ human capital if Inequality 36 is satisfied. (See appendix A for proof).

(36) $\sum_{i=0}^{A_t} h_{i,t}{}^{\gamma} > \frac{1}{1-\gamma}$, $\gamma < 1$

Proposition 1 shows that the amount of other production factors does not impact the firm's decision whether to use human capital. This decision only depends on the value of human capital and its elasticity. At the early stages of the industrialization when the number of types of human capital (A_t) is small, the chance that the inequality is satisfied is low and firms will not invest in human capital in the absence of government intervention. Also, we see that as γ gets closer to 1, the chance that inequality (36) is satisfied becomes smaller. The reason is that the revenue coming from human capital – which is $p_{y,t} L_{Y,t}{}^{\beta} K_{Y,t}{}^{\alpha} \left(\sum_{i=0}^{A_t} h_i{}^{\gamma} \right)$ as shown in appenidx A – grows at rate of $\gamma < 1$, whereas the cost of human capital – which is $\gamma p_{y,t} L_{Y,t}{}^{\beta} K_{Y,t}{}^{\alpha} h_i$ as shown in Appendix D – grows proportionally to the amount of human capital.

Proposition 2. With government intervention, a firm will employ human capital if inequality 37 is satisfied. (See appendix B for proof).

$$(37) \quad \sum_{i=0}^{A_t} h_{i,t}{}^{\gamma} > \frac{1}{1-\gamma(1-s)}$$

Introducing a subsidy will loosen the bounds and will make it possible for the firms to invest in human capital even in the early stages.

If we assume γ is equal to α – as many economists do – the effective range for subsidy should be larger than 20%.

These propositions state that *in the early stages of growth* when the marginal productivity of human capital is low, subsidy will bring firm's profit above the level it would have earned without employing human capital and therefore the firm will choose to employ human capital. While a low subsidy rate will not make enough incentives for firms to employ human capital, a high subsidy rate in a closed economy, increasing the tax burden, will diminish profits. Therefore, in a closed economy only a moderate level of subsidy can shift firms to employ human capital. Also, as we will see in chapter 3, this proposition will help us derive government borrowing in an open economy.

GENERAL EQUILIBRIUM GROWTH

To derive growth at equilibrium, we first consider the supply side and then equate it with growth of the demand side.

Supply Side Growth

As mentioned in the assumptions, the firm's decision-making horizon is short compared to government's. Thus, we start by maximizing the profit for firms in each period and then derive the attainable growth for this case. As shown in Appendix D, Equations (38) to (43) are used to derive growth in terms of the interest rate, factor elasticity, and knowledge profusion factor. These equations mean that: the same amount of each kind of human capital is generated – no index is present on the right-hand side of equation (39). The value of human capital is same for all types and additionally does not vary with time (40). The price of knowledge is the present value of all the future value generate by selling it (42). The marginal productivity of labor is the same across the primary sector and R&D (43).

(38) $$g_{A,t} = \frac{A_{t+1}-A_t}{A_t} = \theta L_{A,t}^{\varphi}$$

(39) $$h_{i,t} = \left[\frac{\gamma^2 p_{y,t} L_{Y,t}^{\beta} K_{Y,t}^{\alpha}}{(1+r)(1-s_{i,t})}\right]^{\frac{1}{1-\gamma}}$$

(40) $$p_{i,t} = \frac{1+r}{\gamma}$$

(41) $$\pi_{i,t} = \left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\gamma^2 p_{y,t} L_{Y,t}^{\beta} K_{Y,t}^{\alpha}}{(1+r)(1-s_{i,t})}\right]^{\frac{1}{1-\gamma}}$$

(42) $$p_{A,t} = \frac{(1+r)\pi_{A,t}}{r}$$

(43) $$MPL_A = MPL_Y$$

As shown in Appendix D, the solution coming out for labor employed in the R&D section is given by Equation 44 where Q is a function of subsidy defined as $Q = \frac{Br(1-s_t)}{\gamma(1-\gamma)\theta\varphi}$.

$$(44) \quad QL_{A,t}^{\varphi-1} + L_{A,t} = L$$

Equation (44) does not have an explicit answer, but it can be solved numerically. In the special case where φ=0.5, however, it becomes a quadratic equation in $L_{A,t}$ with the solution given by (45).

$$(45) \quad \mathrm{L}_{A,t} = \left(\frac{\sqrt{Q^2+4L}-Q}{2}\right)^2$$

Defining $\Lambda = \frac{\alpha}{1-\gamma}$ and $B = \frac{\beta}{1-\gamma}$, and $C_t = \left[\frac{\gamma^2 p_{y,t}}{(1-s_t)(1+r)}\right]^{\frac{1}{1-\gamma}}$, we can rewrite the economy's output as $\mathrm{Y}_t = A_t {C_t}^{\gamma} {L_{Y,t}}^{\mathrm{B}} {K_{Y,t}}^{\Lambda}$

As shown by proposition 4 in Appendix I, the supply side growth is given by Equation (46).

$$(46) \quad \mathrm{g}_{s,t} = \left(1 + \theta {L_{A,t}}^{\varphi}\right)^{\frac{1}{1-\Lambda}} - 1$$

STEADY STATE ANALYSIS ON SUPPLY SIDE

Before looking at the demand side, we show that with the existing constraints, our supply side growth does not converge to a steady-state growth. Note that based on the original CES assumption that α+β+γ=1 we must have Λ+B=1. Defining output and capital intensity as $y_t = {\mathrm{Y}_{Y,t}}/{L_{Y,t}}$ and $\mathrm{k}_t = {\mathrm{K}_t}/{L_{Y,t}}$, we get Equation (47) (see Appendix E for proof) expressing the rate of change in capital intensity ($\dot{\mathrm{k}}_t = \frac{dk}{dt}$) in terms of primary sector's labor growth ($n_{y,t} = \frac{L_t - \mathrm{L}_{A,t}}{L_{t-1} - \mathrm{L}_{A,t-1}}$), technology growth ($\mathrm{g}_{A,t} = {\Delta A_t}/{A_t} = \theta {L_{A,t}}^{\varphi}$), capital depreciation (δ), and the saving rate (σ)

$$(47)\quad \dot{\mathrm{k}}_t = \sigma {C_t}^{\gamma} A_t {\mathrm{k}_t}^{\Lambda} - \left(n_{Y,t} + \delta\right)\mathrm{k}_t$$

This is similar to Frankel (Frankel, 1962). Note that our model has fewer exogenous variables than Frankel's, and specifically exponent Λ and the coefficients are all derived within our model. We can express the growth rate of capital intensity with Equation (48).

$$(48)\quad \frac{\dot{\mathrm{k}}_t}{\mathrm{k}_t} = \sigma {C_t}^{\gamma} A_t {\mathrm{k}_t}^{\Lambda-1} - \left(n_{Y,t} + \delta\right)$$

In Equation (48), three terms, $C_t,\ A_t$, and $n_{Y,t}$, depend on the subsidy level (the first term directly and the other two indirectly via the division of labor across the primary sector and R&D). Therefore, by setting the subsidy level within acceptable bounds, government can effectively adjust the interest and growth rates in the long term. Figure 4 shows the relationship between supply side growth and the subsidy level.

Note that unlike the Solow or Frankel models, the rate of change of capital intensity in our model is dynamic and we cannot make any specific assertion - just based on the information we have – about whether it converges to a steady state or not. As we will see in chapter 3, the limiting factor for growth is the government budget available to fund the subsidies and we will use this limit to find the steady state growth.

GROWTH AT EQUILIBRIUM

On way to find equilibrium is to follow Romer's approach (Romer, 1989) and assume that at equilibrium the supply and demand side growths are equal. Although this does not have to hold in general, it is a special case of the equilibrium that we will present in chapter 3. Using an iso-elastic utility function for infinitely-lived consumers given in Equation (49) and aggregating across identical agents, we derive the Euler equation for the demand side given by Equation (50). In these equations d is the time-preference factor and ρ is the elasticity of substitution (Appendix G shows the derivation).

$$(49)\quad U(C_t) = \sum_{s=t}^{\infty} d^{s-t} \frac{c_s^{1-\rho}}{1-\rho}$$

$$(50)\quad g_{d,t} = [(1 + r_t)d]^{\frac{1}{\rho}} - 1$$

At the equilibrium, the supply and demand sides will grow at the same rate so by equating Equation (48) and (50) we find the growth and interest rate at general equilibrium given by Equation 51 (for this special case).

$$(51)\quad g_{d,m} = g_{s,m} \Rightarrow \left(1 + \theta {L_{A,t}}^{\varphi}\right)^{\frac{1}{1-\Lambda}} = [(1 + r_t)d]^{\frac{1}{\rho}}$$

IMPACT OF GOVERNMENT INTERVENTION ON GROWTH

In Figure 4 we have drawn supply-side growth for three different values of the subsidy, 0, 0.5, and 0.9. We have also drawn the demand-side growth. The intersection of the two specifies the growth at general equilibrium. We observe that when human capital is almost fully subsidized (s=0.9), the growth is maximized. Note that the almost fully subsidized case is an extreme case and is only possible in the early stages of industrialization where size of the R&D sector is small compared to the whole economy.

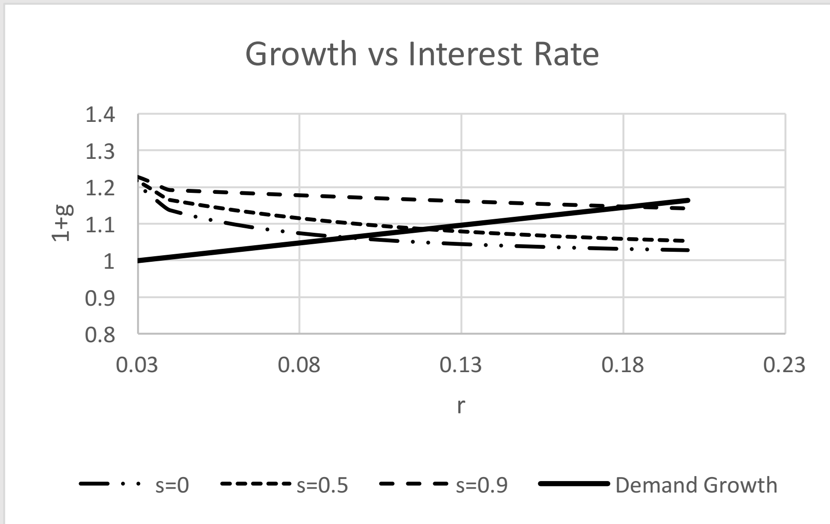

Figure 4. The increasing line shows the growth on the demand side. Dashed decreasing curves show the supply side growth with government intervention. The higher the subsidy, the higher the growth rate. The intersection of the demand and supply side curves specifies the general equilibrium growth.[1]

As the economy grows and the R&D sector becomes a dominant component, government will not be able to fully subsidize human capital since it requires a substantial increase in the tax rate, which will slow down growth. The decision to reduce the subsidy is one of the most important decisions a government aiming at industrialization must make. As mentioned in the introduction, not doing so, and relying on excessive sovereign debt to fund the subsidy, has another adverse effect which is causing fatigue in the domestic economy, where the domestic producers will not try to be self-reliant and operate efficiently. Normally if reaching the level of original income - which would have been reached if the economy did not embark on the industrialization endeavor - takes longer than expected, government faced with middle-class upheaval due to exacerbated income

[11] The parameters used for this graph are as follows: $\beta=0.741$, $\alpha=0.259$, $\theta=0.02$, $\gamma=0.259$, $L=100$, $\varphi=0.5$, $d=0.98$, $\rho=1$.

inequality and high inflation rates will rely on sovereign debt to speed up the growth rate, which may end up in a vicious cycle of increasing debt and finally bankruptcy.

OPTIMUM SUBSIDY AND GROWTH SPEED

Given that the speed of growth is a crucial factor to avoid excessive foreign borrowing, in this section we analyze the speed of growth. In Figure 5, the economy's output is shown for various values of the subsidy. An increasing subsidy has two effects on growth. On the positive side, it increases growth by employing more labor in the R&D sector and generating more technology. On the negative side, having too many workers in the R&D sector, specially in the early stages of industrialization, slows down the production of primary goods - Figure 6[2] shows the labor hires in the R&D sectors for different values of subsidy. One effective policy could be to start industrialization with a lower subsidy rate and gradually increase the rate as the R&D sector grows, finally reducing the subsidy at the final stages of growth to reduce the tax burden on the economy. Note that in the model presented in this section we have assumed the subsidy paid to the R&D sector can be maintained throughout the industrialization. However, as the R&D sector grows, this assumption cannot be maintained without relying in sovereign debt. We look at this in the following chapter.

[2] The following values are used to generate figures 5 to 7: β=0.3, α=0.4, γ=0.3, r=2%, θ=0.01, ϕ=0.5, d=0.93, $P_{Y,t}$=1, ρ=1, and population growth=10%.

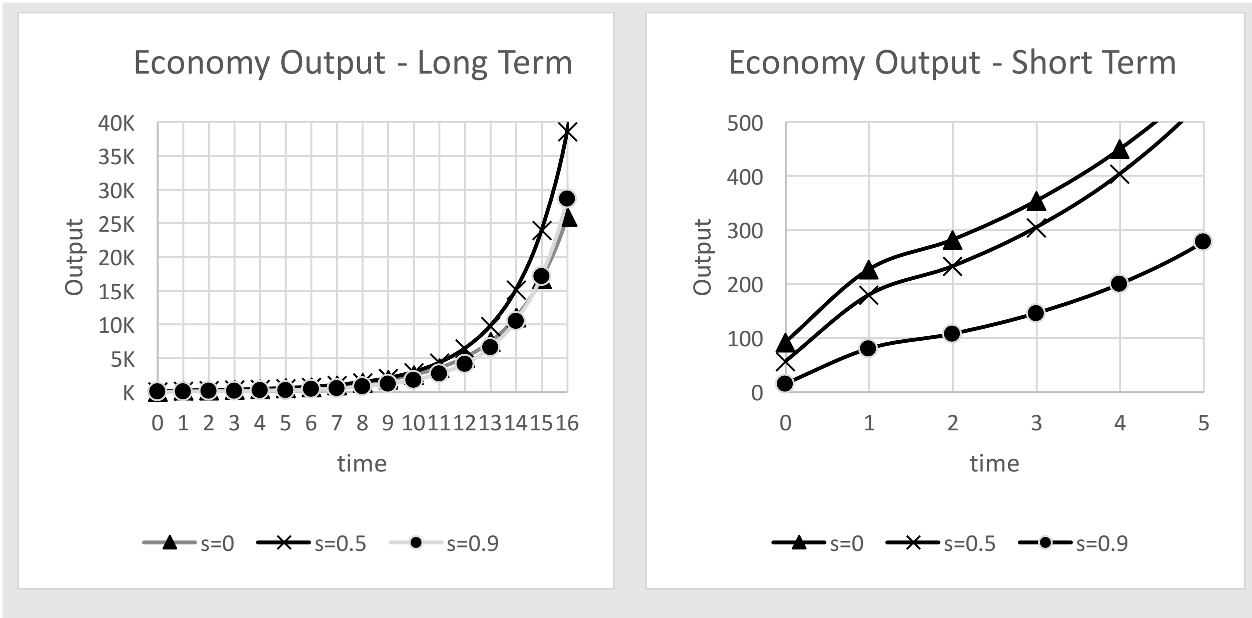

Figure 5. In the left figure, we can compare the economy's output over the long-term for different values of the subsidy. We see that having a high value of subsidy underperforms while a mid-range value for subsidy yields the best growth in the shortest amount of time. On the right, we see that the no-subsidy policy performs better early on.

As shown in Figure 6, as the economy's industrialized sector grows, wages increase at a rate depending on the level of subsidy.

As mentioned in the background section, dualism was the one of the domestic societal reasons for ISI to fail. As shown in Figure 7, this is because industrialization can take several periods to catch up, during which the economy is faced with a substantially lower level of income per capita. Figure 7 shows that although income per capita grows faster with a higher subsidy rate, its level is lower for a higher subsidy level in the early stages of industrialization. To minimize dualism, it is best to pick a mild subsidy policy.

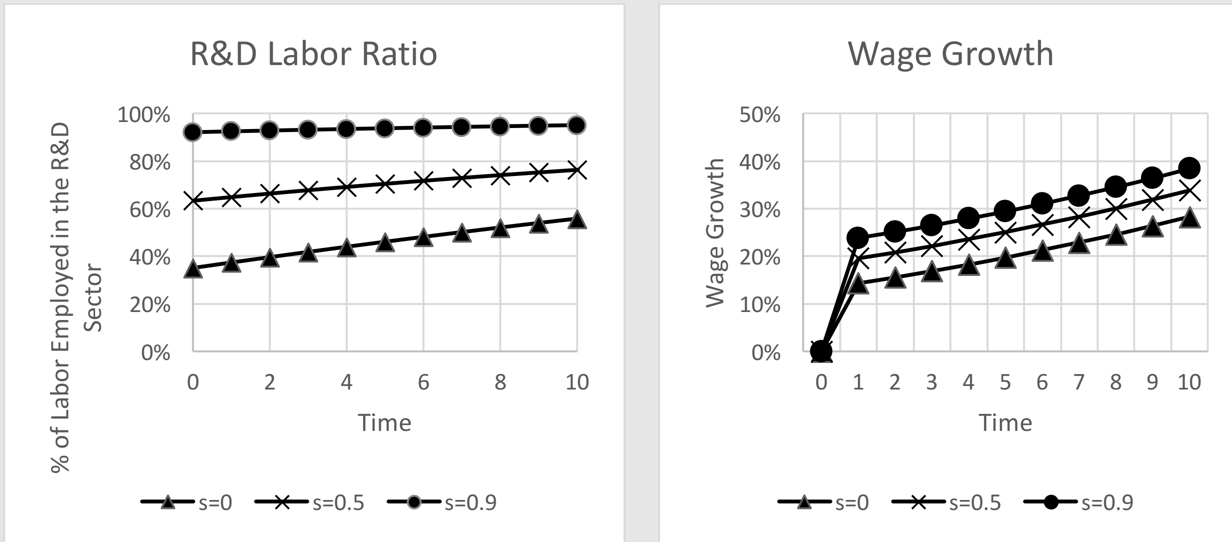

Figure 6. On the left, we see that the labor employed in the R&D sector is higher for a higher subsidy policy. This is because the marginal productivity of labor in the primary sector increases with the subsidy. Therefore, a low-subsidy policy works better in the early stages of industrialization when the R&D sector is not as effective.
On the right, we see that wage growth rate depends on the subsidy level. The higher the subsidy level, the higher the wage growth.

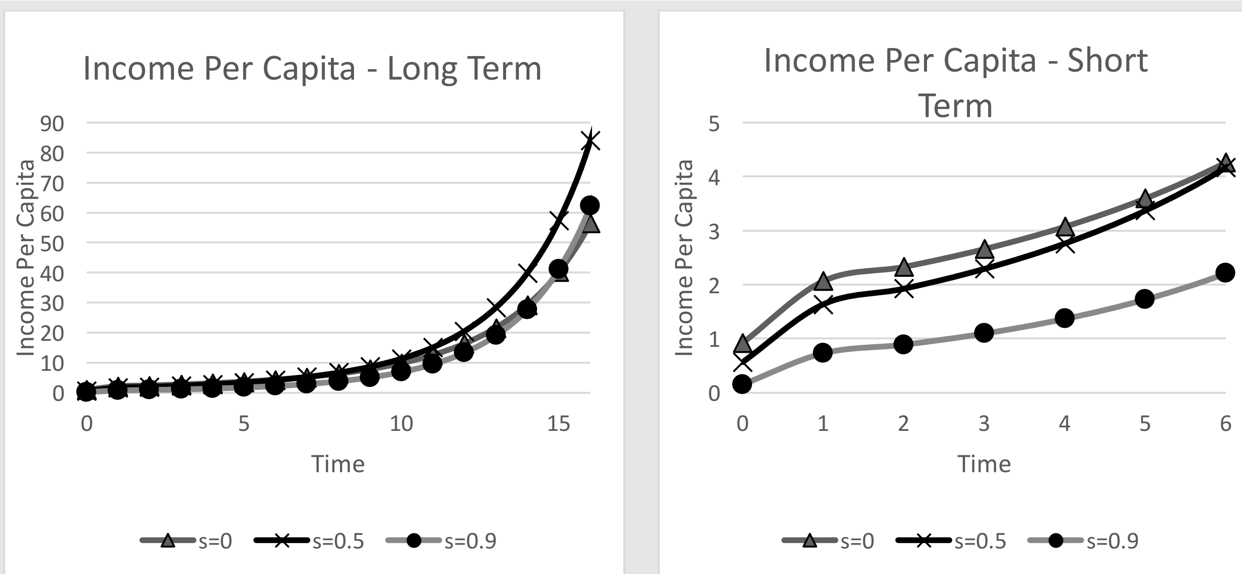

Figure 7. On the left, we see that in the long-term the mild subsidy policy provides a higher income per capita; however, as we see on the right, in the early stages of development, the high and mild subsidy policies lower income per capita, which combined with the high wage growth, causes dualism.

In this chapter, we studied the general equilibrium and observed how different subsidy rates can impact the short and long term macro-economic conditions. Specifically, we saw that a moderate level of subsidization provides the best results in the long-run since not only it creates incentives for employing human capital in production, it does not do it as aggressively as a very high subsidy rate does which results in high employment of production factors in the R&D sector, and depriving the primary sector of these resources. In the next chapter, we consider why resources available in a closed economy are not enough to achieve the maximum level of growth and continue with analyzing the impact of foreign borrowing on growth.

CHAPTER 3

SOVEREIGN DEBT AND GROWTH IN OPEN ECONOMIES

In chapter 2, we specified the general equilibrium values for growth and interest rate at each period. The expressions we saw were functions of the subsidy's rate and we observed that for typical values of production elasticity, a mid-range subsidy maximizes growth. However, we did not elaborate on the source of the subsidy. In a closed economy, as the R&D sector grows and more human capital is used in the primary sector, subsidizing the human capital requires substantial taxation which cannot be sourced from the limited domestic profits made on the primary sectors.

In this chapter, first we derive the growth limit on a closed economy and then compare it with an open economy.

GROWTH LIMITATION IN A CLOSED ECONOMY

First let's look at the maximum attainable growth in a closed economy. We use the model presented in the previous chapter but with a major condition that the subsidy paid to the primary sector is sourced from the same sector. This is shown with Condition 52. The condition specifies that the subsidy paid to the primary sector must be less than the taxes collected.

$$\text{(52)} \quad \sum_{i=0}^{A_t} s_t p_{i,t} h_i < T_t$$

Like Equation 35, we can express the taxes in terms if output minus production expenses.

$$\text{(53)} \quad \mathrm{T}_t = \tau_t \left[{L_{Y,t}}^{\beta} {K_{Y,t}}^{\alpha} \sum_{i=0}^{A_t} {h_i}^{\gamma} - w_t L_{Y,t} - r_t K_{Y,t} - \sum_{i=0}^{A_t} p_{i,t} h_i + \sum_{i=0}^{A_t} s_t p_{i,t} h_i \right]$$

Substituting for production factors and prices from equations 39, 40, and 47, and setting wage and rent to the marginal revenue of labor and capital we obtain Inequality 54 expressing the feasible values for the subsidy in a closed economy (derivation is provided in appendix H).

$$(54)\quad s_{A,t} \leq \frac{(1-\gamma)(1-B)(1-\Lambda)\tau_t}{\gamma + (1-\gamma)(1-B)(1-\Lambda)\tau_t}.$$

We observe that the attainable subsidy rate, $s_{A,t}$, depends on the product elasticities and the tax rate. As shown in Figure 8, the lower the product elasticity of human capital the higher the maximum subsidy rate. This is because as γ moves toward 1, the primary sector's use of human capital goes to infinity and so the government's budget will not be sufficient to fund the subsidy. Using Equation 54 we see that the maximum subsidy level in a closed economy with a typical maximum tax rate of 50% is equal to $(1-B)\,(1-\Lambda)\frac{1-\gamma}{1+\gamma}$. As we saw in chapter 2, a mild range subsidy rate works best to yield a high growth without widely

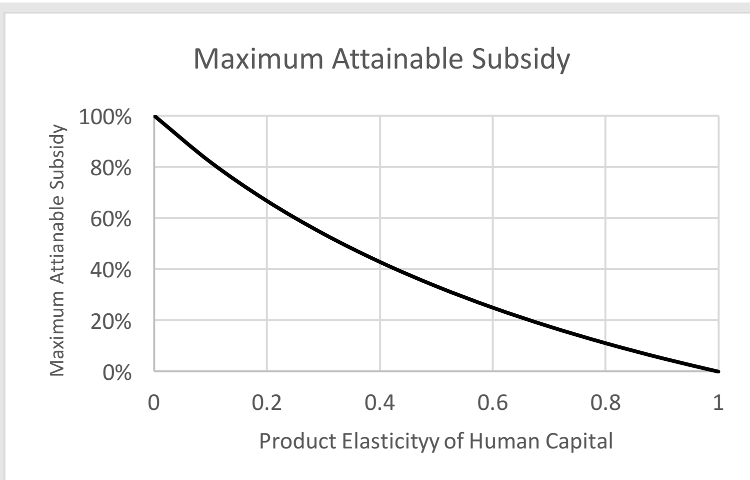

Figure 8. In a closed economy, the maximum attainable subsidy rate is a decreasing function of product elasticity of human capital.

compromising short-term income per capita; however, the maximum attainable subsidy rate is 25% - for highest value of $(1-B)(1-\Lambda)=0.25$ when $B=\Lambda=0.5$.

EXPANDING GROWTH USING SOVEREIGN DEBT

We focus on sovereign debt, which is how the ISI economies attempted to grow in the late 20th century. While we consider a short decision-making horizon for firms, we consider a long-term decision making horizon for the government, which is equal to the entire period of industrialization. The government's goal is to maximize the present value of the output over the entire period of industrialization, which starts with a public announcement of the tax policy that using human capital in production will be subsidized by a given rate. At the same time, the government commits to have the funds available for subsidization by securing a borrowing contract with a foreign bank over the period that industrialization. The government can borrow gradually to minimize the paid interest rate as long as it can pay the subsidies. It also promises to pay off the entire debt by the end of the industrialization term. The government's objective function is provided by Equation 55 and its budget constraint by Equation 56.

$$(55) \quad \chi_t = \max_{s_t, B_t} \sum_{t=0}^{T} \left[\left(\frac{1}{1+r} \right)^t p_{y,t} Y_t \right]$$

$$(56) \quad G_t = T_t + B_{t+1} - B_t - rB_t$$

Equation 56 states that government's spending - which in our case is spent to pay the subsidies - is equal to the taxes plus the changes in borrowing $B_{t+1}-B_t$ minus the interest paid on the current debt rB_t. We can substitute for taxes using the expression given in

Appendix G, and for government spending as total subsidies paid. With some re-arrangement, we obtain Equation (57).

$$(57)\quad C_t A_t L_{Y,t}{}^{B} K_t{}^{\Lambda} = \frac{rB_t - B_{t+1} + B_t}{p_{y,t} C_t{}^{\gamma-1} \tau_t (1-\gamma)(1-B)(1-\Lambda) - s_t \dfrac{1+r}{\gamma}}$$

Using Equations 46 and 57 we get $p_{y,t} Y_t = \frac{rB_t - B_{t+1} + B_t}{\tau_t (1-\gamma)\,(1-B)\,(1-\Lambda) - \frac{s_t \gamma}{1-s_t}}$ and replacing this in Equation 55, we get Equation 58:

$$(58)\quad \chi_t = \max_{s_t, B_t} \sum_{t=0}^{T} \left[\left(\frac{1}{1+r} \right)^{t} \frac{rB_t - B_{t+1} + B_t}{\tau_t (1-\gamma)(1-B)(1-\Lambda) - \dfrac{s_t \gamma}{1-s_t}} \right]$$

Note that χ_t is an increasing function of s_t since the first derivative with respect to subsidy is positive. Therefore, government should use try to maximize its subsidy.

BORROWING POLICIES

As explained in chapter 2, a varying subsidy rate policy will work best for a developing economy. In the early stages, we need to keep the subsidy rate as high as possible to make the subsidy effective. Government can then gradually increase the subsidy rate to speed up the growth but there are two restricting forces not letting the government do so freely. First, as stated in the previous section, government needs to borrow beyond the attainable subsidy rate. Second, a very high subsidy rate has adverse farewell effects by lowering the income per capita.

Based on these limits we can devise separate borrowing policies for the short term and the mid-to-long term.

Also, note that in our model the tax rate is an exogenous variable, and in practice, as discussed in by many scholars, it is set to balance the efficiency costs of collecting taxes and the distributional costs (Slemrod, 1989). Therefore, the borrowing policies adjust the subsidy level to work based on the tax rate determined outside of the subsidy policy.

SHORT-TERM BORROWING POLICIES

We define short term as the periods in which the marginal productivity of human capital is not high enough to make it profitable for firms to employ it. As shown in proposition 3, in the short-term the government must keep the subsidy between γ/(1-γ) and 50% to make it effective.

SHORT-TERM BORROWING POLICY 1, THE "GROWTH-FIRST" POLICY

This first short-term policy only tries to maximize the long-term growth regardless of its welfare effects, therefore we call it the "growth first" policy.

The Growth-First Policy: In the early stages of growth, government will always subsidize human capital formation at the maximum possible rate given by Equation 59.

In the early stages, if the attainable subsidy level is low government borrows to pay beyond its revenue from taxes. This amount is given by Equation 59 for each period, which is the difference between the target subsidy rate, s* and the maximum attainable subsidy times the value of human capital employed by the firms. (See Appendix J for proof).

(59) $$B_t^C = \frac{s^* - s_A^{max}}{1 - s_A^{max}} \gamma p_{y,t} Y_t$$

Note that the maximum level of borrowing is bounded and we will specify its limit soon.

SHORT-TERM BORROWING POLICY 2, THE "GROWTH-AND-DISTRIBUTION" POLICY

As discussed in chapter 2, although the maximum effective subsidy rate maximizes growth in the long term, it adversely affects the income per capita in the short term which can cause dualism, making the country unstable and defeating the industrialization effort altogether. To alleviate the short-term impact, the socialist policy not only borrows to maximize growth, but also to keep the income per capita above a certain limit, denoted by ω. Note that ω is set based on the previous levels of output before the industrialization started. For example, government, can set ω to 75% of last period's output: $\omega = 0.75\ Y_{-1}$

The Growth-and-Distribution Policy: In the early stages of growth, government will always subsidize the maximum effective rate given by Equation (60).

The amount to be borrowed under this policy given by Equation 60.

$$(60) \quad B_t^S = \max\left(\frac{s^* - s_A^{max}}{1 - s_A^{max}} \gamma p_{y,t} Y_t, \omega - Y_t\right)$$

MID-TO-LONG-TERM BORROWING

In the mid to long term, government borrows as much as possible to speed up to growth. Government can borrow only if it can show it can avoid a default on its debt. One of the measures used to assess a government's ability to pay its debt is the ratio of debt to output. The foreign lender will keep lending if the debt to output ratio is below η. As shown in Appendix I, the debt to output ratio dynamics can be expressed using Equation 61.

(61)
$$\frac{B_{t+1}}{Y_{t+1}} = \left[\frac{\left((1+r)d\right)^{\frac{1}{\rho}}}{1+g_{s,t}}\right]\frac{B_t}{Y_t} - \frac{1+g_{s,t}+\left((1+r)d\right)^{\frac{1}{\rho}}}{(1+g_{s,t})(r-g_{s,t})}\left[1+\frac{\Lambda p_y(1-\gamma)(1-B)}{r}-\gamma\right]$$

Assuming labor does not grow substantially during development, we will have constant supply side growth. The economy behaves depending on how supply side growth, $g_s + 1 = (1+\theta {L_A}^{\varphi})^{\frac{1}{1-\Lambda}}$, and the demand side growth rate, $g_d + 1 = [(1+r)d]^{\frac{1}{\rho}}$ compare to each other:

CASE 1. STABLE DEBT TO OUTPUT RATIO

If $g_s > g_d$, the debt to output ratio will be stable and converges to the steady state ξ given by Equation 62. The transition to the steady state is gradual and is shown in Figure 9. Starting from any initial debt-to-output ratio (DOR) like point B, DOR moves toward the steady state point D. In the case shown here, the steady state DOR is within the allowable range set by the lender.

(62)
$$\xi = \frac{1+\frac{\Lambda p_y(1-\gamma)(1-B)g_s}{r}-\gamma}{r-g_s}$$

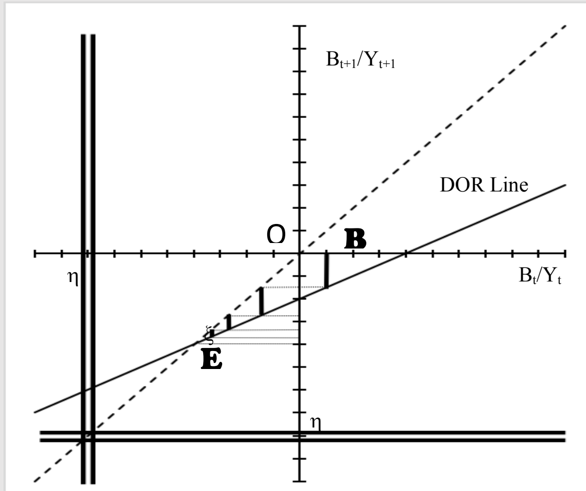

Figure 9. The debt to output ratio ($\frac{B_t}{Y_t}$) gradually converges to steady state. Starting from a positive current account (point B), debt-to-output gradually reaches the negative steady state ratio (point E). The double line shows the max allowed debt-to-output by the lender. In this case since point E is above η, the steady-state is valid.

If the steady state turns out to be outside of the allowable range, the debt-to-output ratio will be capped at η and the government's budget to subsidize the R&D sector will be reduced accordingly, which will decrease the supply side growth. In this case, the we can solve for growth as the intersection of the DOR line and η which is given by Equation 63.

$$(63) \quad g^* = \frac{1-\gamma-\eta r}{\frac{\Lambda p_y(1-\gamma)(1-B)g_S}{r}-\eta}$$

In the stable case, output grows faster than consumption and in steady state the ratio of consumption to output approaches zero. Note that this does not mean that the consumption approaches zero but grows slower than production.

CASE 2. ROMER'S CASE

If $g_{s,t} = g_d = g^*$, we end up with the special case discussed in the previous chapter, which was based on Romer's approach. In this case $B_{t+1} = (1+g^*)B_t$, therefore output and debt will grow at the same rate. In this case economy's transition to steady state is instantaneous and $B_t = \eta Y_0 (1 + g^*)^t$.

CASE 3. TRANSIENT UNSTABLE CASE

If $g_{s,t} < g_{d,t}$, the debt to output ratio will be unstable.

ONE EXAMPLE: OPEN VERSUS CLOSED GROWTH

In this section, we look at some examples to show how our models are applied under closed and open economies. We also look at how different short-term borrowing polices work.

CLOSED ECONOMY

Using the same parameters as in chapter 2[3], and assuming a tax rate of 50%, we see that the maximum attainable subsidy is 13%, which is less than the minimum effective subsidy level of 23%[4]. Therefore, in a closed economy, human capital will never be employed by firms no matter how much subsidy is paid by the government.

OPEN ECONOMY

When the economy opens to the world, in the short-term the government can either apply a capitalist or a socialist borrowing policy. With the former, using Equation (59), government borrows so that the DOR becomes 12%[5]. Using this DOR, government can achieve a short-term subsidy rate of 50%. To calculate the short-term growth, we need to first find the labor allocations to R&D and the primary sector. We start with the second

---

[3] In chapter 2, we used the following values: β=0.3, α=0.4, γ=0.3, r=2%, θ=0.01, ϕ=0.5, d=0.93, $P_{Y,t}$=1, ρ=1, and population growth=10%. Here we have added: τ=50% and change population growth to 0%.

[4] Note that based on proposition 3, the minimum effective subsidy level is equal to γ/(1-γ). For γ=0.3, the minimum becomes 23%

[5] $DOR = B_t^C / Y_t = \frac{0.5 - s_A^{max}}{1 - s_A^{max}} \gamma p_{y,t} = \frac{0.5 - 0.13}{1 - 0.13} 0.3 = 0.12$

period – since the industrialization starts in the first period and first growth is observed in the second period. Using Equation 45, we get: ${L_{A,2}}/{L_2} = 0.67$[6]. Using Equation 46, the growth is calculated as:

$$g_{s,2} = \frac{Y_2}{Y_1} = \left(1 + \theta L_{A,2}{}^{\varphi}\right)^{\frac{1}{1-\Lambda}} - 1 = \left(1 + (0.01)(67^{0.5})\right)^{2.33} - 1 = 20\%$$

Although growth has increased substantially, income per capita is impacted negatively. To see this, we can calculate the output without industrialization with no total factor productivity growth using $Y_t = L_t{}^{\beta} K_t{}^{\alpha}$, while the marginal product of factor is set equal to their prices. We get $K_2 = 5266$ and $Y_2 = 351$. Under industrialization using $Y_t = A_t C_t{}^{\gamma} L_{Y,t}{}^{B} K_{Y,t}{}^{\Lambda}$ to calculate output, and using Appendix I to calculate the capital for the second period, we obtain $K_2 = 2{,}200$ and $Y_2 = 165$. Therefore, income-per-capita has fallen from 3.19 to 1.87. If we do the calculation in a spreadsheet for more periods we see that up to the 10th period, income-per-capita stays below its original level. Given the long time for income-per-capita to surpass its original level, even though the economy is now growing much faster, living standards drops – this is due to the higher wage growth under industrialization. This will exert a lot of pressure on the society, specifically the poor and middle class, who will try to stop the economic overhaul.

Under the growth-and-distribution policy, government tries to maintain a specific level of output. Assume that government decides to keep income-per-capita at least at 75% of its original level. Table 3 shows the required borrowing for the two short-term policies. Up to

[6] We have used $Q = \frac{Br(1-s_t)}{\gamma(1-\gamma)\theta\varphi} = \frac{(0.42)(0.02)(1-0.5)}{(0.3)(0.7)(0.01)(0.5)} = 4$, then using $L_{A,t} = \left(\frac{\sqrt{Q^2+4L}-Q}{2}\right)^2 = \left(\frac{\sqrt{4^2+4(100)}-4}{2}\right)^2 = 67$. Therefore, the ratio is 67%.

the fifth period, the income-per-capita for the developing economy is lower than its original value and so a socialist policy will borrow more to keep it at 75%. From the fifth period on, the economy's output is large enough to maintain the required level of income per capita and government reverts to the growth-first policy.

Table 3. Borrowing under capitalist and socialist policies. Up to the 5th period, the amount to borrow to keep income per capital at 75% of its original value requires higher borrowing.

Period	Growth-First Debt	Growth First DOR	Growth-and-Dist. Debt	Growth-and-Dist. DOR
1	6.68	0.12	186.86	3.55
2	20.82	0.12	99.62	0.60
3	26.13	0.12	84.20	0.40
4	33.18	0.12	57.76	0.22
5	42.64	0.12	15.24	0.04

WHY DEFAULT HAPPENS

There are many factors that can derail the economy from the perfect path we have looked at so far. Specifically, for the Latin American countries, we can list the ones that have been more evident; First, ISI's major goal was to produce the primary goods consumed domestically inside the country. Given that there was very low competition, given the lack of imports, this inward oriented-ness resulted in poor quality products that were not demanded by foreign consumers. Second, in 1979 Paul Volcker was appointed by Carter to head the Federal Reserve Bank and he decided to increase the interest rates to fight the high inflation rate. The increase in interest rates caused the dollar to appreciate against foreign currencies, including Mexican and Argentinian Pesos, and Brazilian Real. Had ISI economies produced at high quality, they could have benefited from their weak currency by increasing their exports and adding more to their foreign assets. In this section, we try to use our models and see how the combination of these two factors can cause a default.

MODELING DEFAULT

Equation (62) shows that, as long as the economy's debt-to-output ratio (DOR) stays in the steady state, DOR is an increasing function of growth. Therefore, ***the higher growth an economy has, the higher DOR it will sustain***.

Also, we see that if the interest rate increases to the point where $1 + g_{s,t}$ is still greater than $\left((1+r)d\right)^{\frac{1}{\rho}}$, the DOR remains stable and is a decreasing function of the interest rate. Assume that at period u, the interest rate of the lending economy is raised from its original level r_1 to r_2, where $1 + g_{s,t} > \left((1+r_2)d\right)^{\frac{1}{\rho}}$: this will reduce the steady state DOR. Using Equation 62 we can calculate the change in DOR, which is given by Equation 64.

$$\text{(64)} \quad \Delta DOR = \frac{(r_2-r_1)\left(1-\gamma+\frac{r_2+r_1-g}{r_2 r_1}V\right)}{(r_2-g)(r_1-g)}$$

If at period u the economy has already reached the previous steady state of its DOR, government must use a portion of its spending equal to ΔDOR to repay the debt and bring down its DOR to the new steady state. This will reduce the level of subsidy, lowering the level of human capital used in the R&D sector, reducing the amount of labor employed by the R&D sector which will in turn reduces

the level of growth. When growth is decreased to the new level, again based on Equation 62 – which is an increasing function of g – the steady state DOR is reduced even more. As shown in Figure 10, the economy enters a vicious cycle where DOR and growth keep shrinking repeatedly. At any point during the vicious cycle if the government's budget is not enough to repay the debt, it will end up in default.

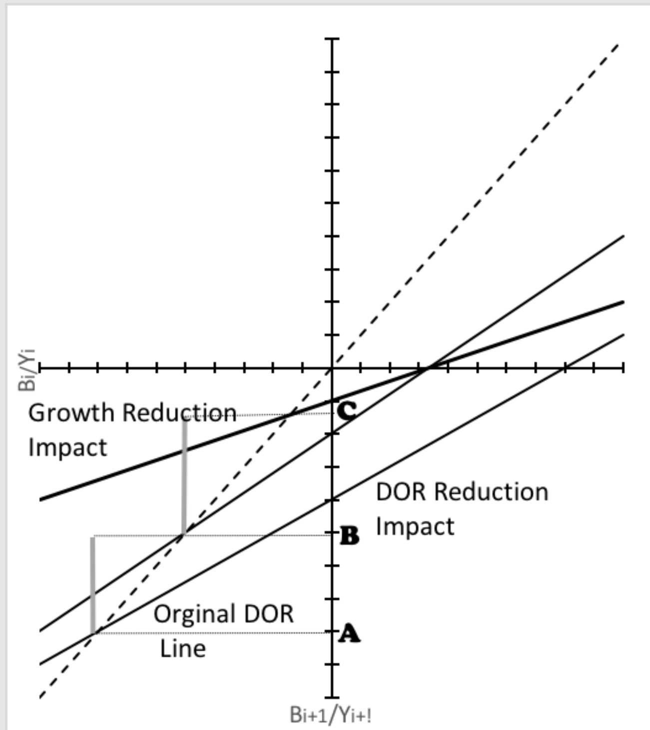

Figure 10. When the interest rate is increased, DOR will enter a vicious cycle where DOR and growth push each other down repeatedly. In this figure, DOR was initially at the steady state point A. The increase in interest rate pushes DOR to the new steady state B lowering the growth which will lower DOR even further to steady state point C.

Production Quality Impact

Now we consider what would have happened if the quality of production were high, and hence primary goods could have been exported more upon a depreciation of local currency. After the lending country's interest rate increases to r_2, the local currency depreciates compared to the foreign currency, and the price of imported primary goods drops depending on the exchange rate pass-through. At the same time, the volume of exports grows based on the elasticity of demand for the imported primary good. In fact, if the quality of primary goods were very poor, they could be considered as inferior, and foreign countries would import less of them at lower prices. Showing exchange rate pass through with ψ, the price elasticity of demand for primary goods in the importing country with λ, and the exchange rate at a given interest rate by $\epsilon(r_t)$, the economy's change in net foreign assets when the interest rate rises from r_1 to r_2 is given by Equation 67 (derivation is provided in Appendix K).

$$\text{(66)} \quad \Delta B_u = \left(\psi\epsilon(r_2)P_{y,u}\right)^{1-\lambda} - \left(\psi\epsilon(r_1)P_{y,u}\right)^{1-\lambda}$$

This increase in net foreign assets can be used to repay the debt and minimize the impact on domestic growth. If the goods have high quality, their demand will be elastic ($\lambda > 1$) and therefore a decrease in prices $\psi\epsilon(r_2)P_{y,u} < \psi\epsilon(r_1)P_{y,u}$ will provide a positive net foreign asset. For unit-elastic goods ($\lambda = 1$), net foreign assets do not change, and for low quality inelastic goods, net foreign assets decrease when interest rate increases.

In this chapter, we presented the open economy model and observed that relying on foreign borrowing increases the maximum attainable subsidy rate, and hence increasing the growth rate. We considered different borrowing polices that the government can apply to consider not only the final growth level but also the welfare of the society throughout the growth period. Finally, we studied the open-economy general equilibrium generalizing the closed-economy equilibrium presented in the previous chapter and used it to study the maximum debt-to-output ratio viable for an economy.

CONCLUSION

The developing countries' debt crisis of the1980s shows that when countries which protect their industry by accumulating debt without creating efficient export-oriented industries will eventually end up in default. This familiar pattern happens whenever excessive funds are attracted by economies without the capacity or the required supervision to employ them in efficient investments for increasing long-term growth and exports. Usually both the lender and the creditor refuse to realize the crisis until the disastrous impacts completely unfold. At this stage, the debtor would have a long and severe path to recovery, and would face major political turmoil and difficult economic austerity measures. As for the creditors, a partial repayment and restructuring plan is the best they can hope for.

Foreign borrowing, if managed prudently, will be beneficial for both the lender and the borrower. It can provide a higher return on investment for the lender while improving the socioeconomic state of the borrower. But this is highly conditioned on the prudent action of both the parties involved. It is up to the lender to make sure it considers its own capital requirement and the risk exposed by the debt ratios of the borrowing country. The borrower should use the funds to increase its long-term capacity to produce at world-level quality, increase its domestic growth, and guarantee its debt obligations.

FUTURE RESEARCH

This study makes a major contribution by creating a model and using it to specify the optimum amount of government policy toward the R&D sector in a closed economy. However, this study does not consider other important economic factors that have a considerable impact on growth including inflation, distributional effects, currency destabilization, and recessionary periods caused by external shocks. These parameters, if not controlled, can have an adverse feedback effect that impedes growth and can cause sovereign debt default. In future research, I will expand this model to include those parameters and devise methods to control them within safe ranges. I will also consider how policy makers can tune their monetary and fiscal tools to control the speed and pattern with which their economies reach the desired level of growth.

While this thesis considers a positive scenario in which the industrializing economy can execute per its plan, there are always chances of facing unforeseen shocks.

APPENDICES

APPENDIX A. PROOF FOR PROPOSITION 1

Firms will only employ human capital if they can buy it for less than the gains from using them in production. Note that this is a loose bound since both labor and capital employed in production will be less when firms use human capital. Part of the labor will be employed in the R&D sector and firms have less budget to employ the same level of capital.

$$p_{y,t} {L_{Y,t}}^{\beta} {K_{Y,t}}^{\alpha} \left(\sum_{i=0}^{A_t} {h_i}^{\gamma} - 1 \right) > \sum_{i=0}^{A_t} p_i h_i$$

As shown in Appendix D, firms choose the amount of human capital maximizing their profit which we can use to express the price based on production factors.

$$p_i = \gamma p_{y,t} {L_{Y,t}}^{\beta} {K_{Y,t}}^{\alpha} {h_i}^{\gamma-1}$$

Using these two equation to eliminate price and simplifying we get:

$$\sum_{i=0}^{A_t} {h_i}^{\gamma} > \frac{1}{1-\gamma}$$

APPENDIX B. PROOF FOR PROPOSITION 2

Proof is like Appendix A with the difference that firms are only paying for the unsubsidized part of the human capital:

$$p_{y,t} {L_{Y,t}}^{\beta} {K_{Y,t}}^{\alpha} \left(\sum_{i=0}^{A_t} {h_i}^{\gamma} - 1 \right) > \sum_{i=0}^{A_t} (1 - s_i) p_i h_i$$

Following the same approach, we get:

$$\sum_{i=0}^{A_t} {h_i}^{\gamma} > \frac{s}{s(1+\gamma)-s}$$

Appendix D. Derivation of Supply side Growth

As we explained in the assumptions, firms maximize their profit for each period separately with respect to production factors. If capital completely depreciates in each period, the profit function for the firm is given by:

$$\pi_t = (1-\tau_t)\left[p_{y,t}L_{Y,t}^{\beta}K_{Y,t}^{\alpha}\sum_{i=0}^{A_t} h_{i,t}^{\gamma} - r_t K_{Y,t} - w_t L_{Y,t} + \sum_{i=0}^{A_t}\left(s_{i,t}-1\right)p_{i,t}h_{i,t}\right]$$

The first order condition for human capital is given as:

$$\frac{\partial \pi_t}{\partial h_{i,t}} = 0 \Rightarrow (1-\tau_t)\left[\gamma p_{y,t}L_{Y,t}^{\beta}K_{Y,t}^{\alpha}h_{i,t}^{\gamma-1} + \left(s_{i,t}-1\right)p_{i,t}\right] = 0$$

$$\Rightarrow\ p_{i,t} = \frac{\gamma p_{y,t}L_{Y,t}^{\beta}K_{Y,t}^{\alpha}h_{i,t}^{\gamma-1}}{1-s_{i,t}}$$

The intermediate firms producing human capital also try to maximize their profit. These firms generate a unit of human capital in the current period and sell it to the primary firms in the next period at the above price. Considering interest rate, we can write the profit for producing and selling each type of human capital as:

$$\pi_{i,t} = \frac{p_{i,t}h_{i,t}}{(1+r)} - h_{i,t}$$

Substituting for $p_{i,t}$ the first order condition becomes:

$$\frac{\partial \pi_{i,t}}{\partial h_{i,t}} = 0 \ \Rightarrow \ \frac{\gamma^2 p_{y,t} L_{Y,t}^{\beta} K_{Y,t}^{\alpha} h_{i,t}^{\gamma-1}}{(1+r)\left(1-s_{i,t}\right)} = 1$$

$$\Rightarrow h_{i,t} = \left[\frac{\gamma^2 p_{y,t} L_{Y,t}^{\beta} K_{Y,t}^{\alpha}}{(1+r)\left(1-s_{i,t}\right)}\right]^{\frac{1}{1-\gamma}}$$

Substituting $h_{i,t}$ back in the expression for $p_{i,t}$ we get:

$$\mathrm{p}_{i,t} = \frac{1+r}{\gamma}$$

Since the price of human capital is same for all types and does not vary with time we will denote $\mathrm{p}_{i,t}$ with p from now on. Substituting for $h_{i,t}$ and $\mathrm{p}_{i,t}$ the profit for the intermediate sector becomes:

$$\pi_{i,t} = \left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\gamma^2 p_{y,t} L_{Y,t}^{\beta} K_{Y,t}^{\alpha}}{(1+r)\left(1-s_{i,t}\right)}\right]^{\frac{1}{1-\gamma}}$$

Since we have assumed that R&D sector has a monopoly in licensing the knowledge it produces and faces a perfect competition in the intermediate market, it sets the price of knowledge to the present value of all the future profit made by the intermediate firms.

$$p_A = P.V.(\,\pi_{i,t}) = \sum_{s=t}^{\infty} \frac{\pi_{i,t}}{(1+r)^{s-t}} = \frac{(1+r)\pi_{i,t}}{r} = (\frac{1+r}{r})\left(\frac{1-\gamma}{\gamma}\right)\left[\frac{\gamma^2 p_{y,t} L_{Y,t}^{\beta} K_{Y,t}^{\alpha}}{(1+r)\left(1-s_{i,t}\right)}\right]^{\frac{1}{1-\gamma}}$$

Knowing the prices, we can now try to find the amount of labor employed in the R&D and primary sectors. We do this by equating the marginal revenue of labor in the two sectors:

$$MRL_A = MRL_Y \Longrightarrow \frac{\partial}{\partial L_{A,t}} p_A \Delta A_t = \frac{\partial}{\partial L_{Y,t}} p_{y,t} L_{Y,t}^{\beta} K_{Y,t}^{\alpha} \sum_{i=0}^{A_t} h_{i,t}^{\gamma}$$

Substituting for p_A and $h_{i,t}$ and using the knowledge generation process $A_{t+1} - A_t = \Delta A_t = \theta L_{A,t}{}^{\varphi} A_t$ we get:

$$L_{Y,t} = \frac{(1 - s_{i,t})Br}{\gamma(1-\gamma)\theta\varphi} L_{A,t}{}^{1-\varphi}$$

Replacing $L_{Y,t}$ in $L_{Y,t} + L_{A,t} = L$ we obtain Equation 44 in Chapter 2.

APPENDIX E. SUPPLY SIDE STEADY-STATE ANALYSIS

Differentiating $\mathrm{k}_t = \mathrm{K}_t / L_{Y,t}$ we get:

$$\dot{\mathrm{k}_t} = \frac{\dot{K_t} L_{Y,t} - \dot{L_{Y,t}} \mathrm{K}_t}{L_{Y,t}{}^2}$$

$$= \left(\frac{\dot{K_t}}{L_{Y,t}}\right) - \left(\frac{\mathrm{K}_t}{L_{Y,t}}\right)\left(\frac{\dot{L_{Y,t}}}{L_{Y,t}}\right)$$

$$= \frac{\dot{K_t}}{L_{Y,t}} - n_{Y,t}\mathrm{k}_t$$

If capital depreciates at rate δ and output is saved as rate σ we will have:

$$\dot{K_t} = \sigma \mathrm{Y}_t - \delta \mathrm{K}_t$$

Replacing $\dot{K_t}$ into the previous equation we get:

$$\dot{\mathrm{k}_t} = \frac{\sigma \mathrm{Y}_t}{L_{Y,t}} - \frac{\delta \mathrm{K}_t}{L_{Y,t}} - n_{Y,t}\mathrm{k}_t$$

Also, we have:

$$\frac{\mathrm{Y}_t}{L_{Y,t}} = \frac{C_t{}^{\gamma} A_t L_{Y,t}{}^{1-\Lambda} K_t{}^{\Lambda}}{L_{Y,t}} = C_t{}^{\gamma} A_t \left(\frac{\mathrm{K}_t}{L_{Y,t}}\right)^{\Lambda} = C_t{}^{\gamma} A_t \mathrm{k}_t{}^{\Lambda}$$

Combining the last two equations we get:

$$\dot{\mathrm{k}}_t = \sigma \mathrm{y}_t - \left(n_{Y,t} + \delta\right)\mathrm{k}_t = \sigma C_t{}^{\gamma} A_t \mathrm{k}_t{}^{\Lambda} - \left(n_{Y,t} + \delta\right)\mathrm{k}_t$$

APPENDIX F. DERIVATION OF DEMAND SIDE GROWTH (ALTERNATIVE APPROACH)

For the special case that ϕ=1 and total labor is fixed, the division of labor among R&D and the primary sector will also stay fixed. Therefore, we will have:

$$A_m = (1 + \theta L_A)^m A_0$$

We can derive growth in period *m* as below:

$$1 + g_{s,m} = \frac{\mathrm{Y}_m}{\mathrm{Y}_{m-1}} = (1 + \theta L_A)\left(\frac{\mathrm{K}_m}{\mathrm{K}_{m-1}}\right)^{\Lambda}$$

If there is no depreciation, we will have $\mathrm{K}_m = s\mathrm{Y}_{m-1}$. Replacing for Y_{m-1} recursively to Y_0 we can show that:

$$\mathrm{K}_m = [\sigma C_t{}^{\gamma}(1 + \theta L_A) A_0 Q^B]^{\frac{1-\Lambda^m}{1-\Lambda}} \mathrm{K}_0{}^{\Lambda^m}$$

Combining the last two equation we derive the expression given by Equation 48.

APPENDIX G. DERIVATION OF SUPPLY SIDE GROWTH

On the demand side, the level of consumption is determined by maximizing the present value of the future utilities under the current account constraint:

$$\max_{c_s} U_t = \max_{c_s} \sum_{s=t}^{\infty} d^{s-t} \frac{C_s^{1-\rho}}{1-\rho}$$

$$s.t. (1+r)B_t = C_t + G_t + I_t - Y_t + B_{t+1}$$

The first order condition with respect to consumption is given by:

$$u'(C_t) = (1+r)du'(C_{t+1})$$

For an iso-elastic utility $u(C_t) = \sum_{s=t}^{\infty} d^{s-t} \frac{c_s^{1-\rho}}{1-\rho}$, $u'(C_t) = (1+r) = d^t C_t^{-\rho}$. Therefore, we have:

$$d^t C_t^{-\rho} = (1+r)d^{t+1}C_{t+1}^{-\rho}$$

$$\Rightarrow C_{t+1} = [(1+r)d]^{\frac{1}{\rho}} C_t$$

$$\Rightarrow g_{d,t} \triangleq \frac{C_{t+1}}{C_t} - 1 = [(1+r)d]^{\frac{1}{\rho}} - 1$$

APPENDIX H. DERIVATION OF FEASIBLE SUBSIDY AND TAX IN A CLOSED ECONOMY

Using definitions of Λ, B, C_t from chapter 2, we can rewrite Equation 53 as

$$\mathrm{T}_t = \tau_t\left[p_{y,t}{C_t}^{\gamma} A_t {L_{Y,t}}^{B} {K_t}^{\Lambda} - w_t L_{Y,t} - r_t K_t - \gamma p_{y,t}{C_t}^{\gamma} A_t {L_{Y,t}}^{B} {K_t}^{\Lambda}\right]$$

We assume firm first decides on how much labor it employs and then decides on its optimal level of capital. This order makes the capital level dependent on the labor level – we could have assumed the reverse order and still yield the same result here but for other purposes, the order can make a difference. Using definition of wage as the marginal revenue of labor we get:

$$\frac{\partial \pi_{i,t}}{\partial L_{y,t}} = 0 \Rightarrow p_{y,t} A_t C_t^{\gamma} B L_{Y,t}^{B-1} K_t^{\Lambda} - w_t - \gamma p_{y,t} B C_t^{\gamma} A_t L_{Y,t}^{B-1} K_t^{\Lambda} = 0$$

$$\Rightarrow w_t = p_{y,t} A_t B L_{Y,t}^{B-1} K_t^{\Lambda} C_t^{\gamma} (1-\gamma)$$

Substituting for w_t and using the definition of rental rate of capital as the marginal revenue of capital we get:

$$\frac{\partial \pi_{i,t}}{\partial K_t} = 0 \Rightarrow p_{y,t} A_t C_t^{\gamma} \Lambda L_{Y,t}^{B} K_t^{\Lambda-1} (1-\gamma)(1-B) - r_t = 0$$

$$\Rightarrow r_t = p_{y,t} A_t \Lambda L_{Y,t}^{B} K_t^{\Lambda-1} C_t^{\gamma} (1-\gamma)(1-B)$$

Therefore, we can write the tax expression as:

$$\mathrm{T}_t = \tau_t p_{y,t} C_t^{\gamma} A_t L_{Y,t}^{B} K_t^{\Lambda} (1-\gamma)(1-B)(1-\Lambda)$$

Substituting this in Condition 52 we get:

$$s_t A_t C_t L_{Y,t}^{B} K_t^{\Lambda} \frac{1+r}{\gamma} \leq \tau_t p_{y,t} C_t^{\gamma} A_t L_{Y,t}^{B} K_t^{\Lambda} (1-\gamma)(1-B)(1-\Lambda)$$

$$\Rightarrow \frac{1+r}{\gamma} s_t \leq \tau_t p_{y,t} C_t^{\gamma-1} (1-\gamma)(1-B)(1-\Lambda)$$

$$\Rightarrow \frac{s_t}{1-s_t} \leq \frac{(1-\gamma)(1-B)(1-\Lambda)}{\gamma} \tau_t$$

APPENDIX I. DERIVATION OF GROWTH AND DEBT AT EQUILIBRIUM

As shown in Appendix H, for a firm maximizing its profit first by hiring labor and then by employing capital we have – We have defined $E_t = A_t L_{Y,t}^{B} C_t^{\gamma}$. We also assume that interest rate and output price are fixed for the duration of development.

$$r_t = E_t \Lambda p_y K_t^{\Lambda-1}(1-\gamma)(1-B)$$

$$\Rightarrow K_t = \left[\frac{E_t \Lambda p_y (1-\gamma)(1-B)}{r}\right]^{\frac{1}{1-\Lambda}}$$

Therefore, we can express output as:

$$Y_t = E_t \left[\frac{E_t \Lambda p_y (1-\gamma)(1-B)}{r}\right]^{\frac{\Lambda}{1-\Lambda}} = E_t^{\frac{1}{1-\Lambda}} \left[\frac{\Lambda p_y (1-\gamma)(1-B)}{r}\right]^{\frac{\Lambda}{1-\Lambda}}$$

Defining $1 + g_{s,t} = \left(1 + \theta L_{A,t}{}^{\varphi}\right)^{\frac{1}{1-\Lambda}}$, we have:

$$E_{t+1} - E_t = L_{Y,t}{}^{B} C_t{}^{\gamma}(A_{t+1} - A_t) = L_{Y,t}{}^{B} C_t{}^{\gamma} A_t \theta L_{A,t}{}^{\varphi} = E_t \theta L_{A,t}{}^{\varphi}$$

$$\Rightarrow E_{t+1} = \left(1 + \theta L_{A,t}{}^{\varphi}\right) E_t = \left(1 + g_{s,t}\right)^{1-\Lambda} E_t$$

Proposition 4. Assuming a fixed interest rate and a fixed output price for the duration of development the supply side growth is indeed equal to $g_{s,t}$.

Proof: Using the definition of supply side growth, we have (where we have used the above equations to for E_{t+1} and K_t).

$$Supply\ Side\ Growth \triangleq \frac{Y_{t+1}}{Y_t} - 1 = \frac{E_{t+1} K_{t+1}{}^{\Lambda}}{E_t K_t{}^{\Lambda}} = g_{s,t}$$

We can also specify investment and government spending in terms of Y_t and $g_{s,t}$:

$$I_t = K_{t+1} - K_t = \left[\frac{\Lambda p_y (1-\gamma)(1-B)}{r}\right]^{\frac{1}{1-\Lambda}} \left(E_{t+1}{}^{\frac{1}{1-\Lambda}} - E_t{}^{\frac{1}{1-\Lambda}}\right)$$

$$= \left[\frac{\Lambda p_y(1-\gamma)(1-B)}{r}\right]^{\frac{1}{1-\Lambda}} \left(1 + g_{s,t} - 1\right) E_t^{\frac{1}{1-\Lambda}}$$

$$= \left[\frac{\Lambda p_y(1-\gamma)(1-B)}{r}\right]^{\frac{1}{1-\Lambda}} g_{s,t} E_t^{\frac{1}{1-\Lambda}}$$

$$= \left[\frac{\Lambda p_y(1-\gamma)(1-B)}{r}\right] g_{s,t} Y_t$$

Also:

$$G_t = \gamma C_t{}^{\gamma} A_t L_{Y,t}{}^{B} K_t{}^{\Lambda} = \gamma Y_t$$

We can replace Y_t, I_t, and G_t into the fundamental current account equation to get a first order difference equation for debt to output ratio, $W_t = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_t - I_t - G_t)$ and $\nu = 1 - [(1+r)d]^{\frac{1}{\rho}}$:

$$CA_t = B_{t+1} - B_t = Y_t - I_t - G_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_t - I_t - G_t) - \frac{\nu}{1+r}W_t$$

$$= -\nu B_t - \frac{g_{s,t}+\nu}{r-g_{s,t}}\left[1 + \frac{\Lambda p_y(1-\gamma)(1-B)}{r} - \gamma\right] Y_t$$

$$\Rightarrow \frac{B_{t+1}}{Y_{t+1}} = \left[\frac{\left((1+r)d\right)^{\frac{1}{\rho}}}{1+g_{s,t}}\right]\frac{B_t}{Y_t} - \frac{1+g_{s,t}+\left((1+r)d\right)^{\frac{1}{\rho}}}{(1+g_{s,t})(r-g_{s,t})}\left[1 + \frac{\Lambda p_y(1-\gamma)(1-B)}{r} - \gamma\right]$$

Appendix J. Derivation of Short-Term Borrowing

The amount of borrowing it the difference of maximum effective subsidy which is equal to 0.5 - based on proposition 3 - and the maximum attainable subsidy level times the volume of human capital being employed:

$$B_t = (0.5 - s_A^{max})ph_t = (0.5 - s_A^{max})\frac{1+r}{\gamma}C_t A_t {L_{Y,t}}^{B} {K_t}^{\Lambda}$$

$$= p_{y,t}(0.5 - s_A^{max})\frac{1+r}{\gamma}{C_t}^{1-\gamma}{C_t}^{\gamma} A_t {L_{Y,t}}^{B} {K_t}^{\Lambda}$$

$$= p_{y,t}(0.5 - s_A^{max})\left(\frac{1+r}{\gamma}\right)\left[\frac{\gamma^2 p_{y,t}}{(1+r)(1-s_A^{max})}\right]Y_t$$

$$= \frac{0.5 - s_A^{max}}{1 - s_A^{max}}\gamma p_{y,t} Y_t$$

Appendix K. Derivation of Change in Foreign Assets

Using the definition of price elasticity of demand, we have:

$$\lambda = -\left(\frac{dQ}{Q}\right)\left(\frac{P_{y,t}^*}{dP_{y,t}^*}\right)$$

$$\Rightarrow \frac{dQ}{Q} = -\lambda\frac{dP_{y,t}^*}{P_{y,t}^*}$$

$$\Rightarrow \log Q = -\lambda \log P_{y,t}^*$$

$$\Rightarrow Q = {P_{y,t}^*}^{-\lambda}$$

Total revenue of imports can be calculated as:

$$R = P^*_{y,t}.\,Q = {P^*_{y,t}}^{1-\lambda}$$

Changes in the foreign assets due to exports is equal to the total revenue change due to change in the exchange rate. Assuming exchange rate changes from $\epsilon(r_1)$ to $\epsilon(r_2)$ we have, using the definition of exchange rate pass through we have:

$$\Delta B_u = \Delta R = \left(\psi\epsilon(r_2)P_{y,u}\right)^{1-\lambda} - \left(\psi\epsilon(r_1)P_{y,u}\right)^{1-\lambda}$$

Appendix L. Simulation Sensitivity Analysis

In this appendix, we revisit our simulations for different values of parameters.

1. **Output growth sensitivity to the interest rate**: in the simulations presented in Figures 5, we used *r=2%*. Figure 11 shows growth as a function of interest rate. The Y axis shows interest rate varying from 1% to 20% and the X axis shows growth at the 10th and 20th periods for three different subsidy rate: 0, 20% and 50%. We see that when interest rate drops growth increases in all cases and stay highest for a subsidy rate of 50%.

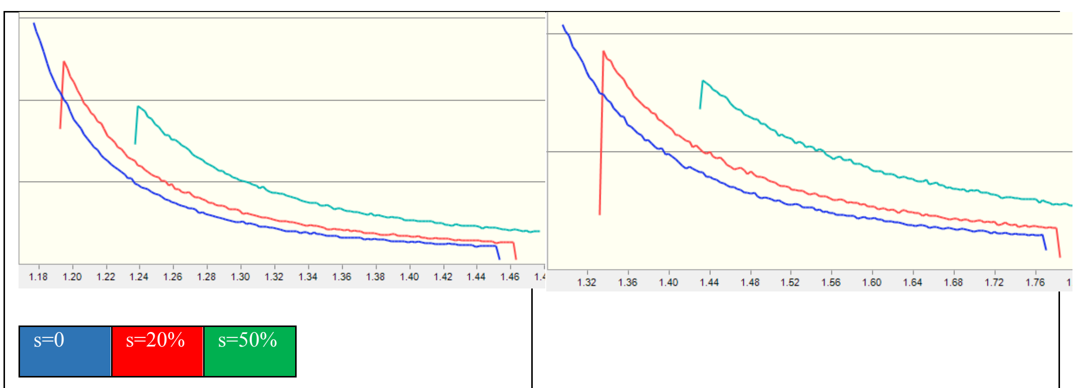

Figure 11. Sensitivity to interest rate for output growth. The Y axis shows the interest rate varying from 1% to 20%. X axis on the left shows growth at the 10th period and on the right growth at 20th period.

2. **Output growth sensitivity to product elasticity of human capital**: in the simulations presented in Figures 5, we used *γ=0.3.* Figure 12 shows growth as a function of γ. The Y axis shows γ varying from 0.1 to 0.3 and the X axis shows growth at the 10th and 20th periods for three different subsidy rate: 0, 20% and 50%. We see that growth in all cases is highest for a subsidy rate of 50%.

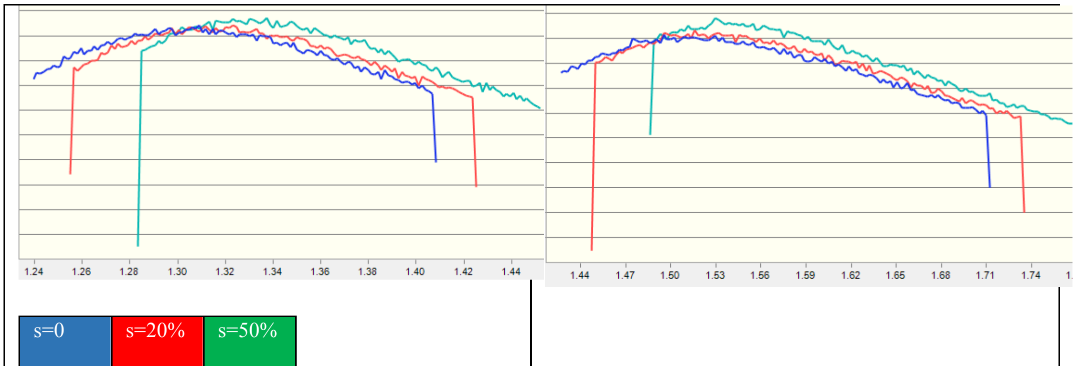

Figure 12. Sensitivity to product elasticity of human capital for output grow. The Y axis shows gamma varying from 0.1 to 0.3. X axis on the left shows growth at the 10th period and on the right growth at 20th period.

3. **Wage growth sensitivity to the interest rate**: in the simulations presented in Figures 6, we used *r=2%.* Figure 13 shows growth as a function of interest rate. The Y axis shows interest rate varying from 1% to 20% and the Y axis shows growth at the 10th and 20th periods for three different subsidy rate: 0, 20% and 50%. We see that when interest rate drops growth increases in all cases and stay highest for a subsidy rate of 50%.

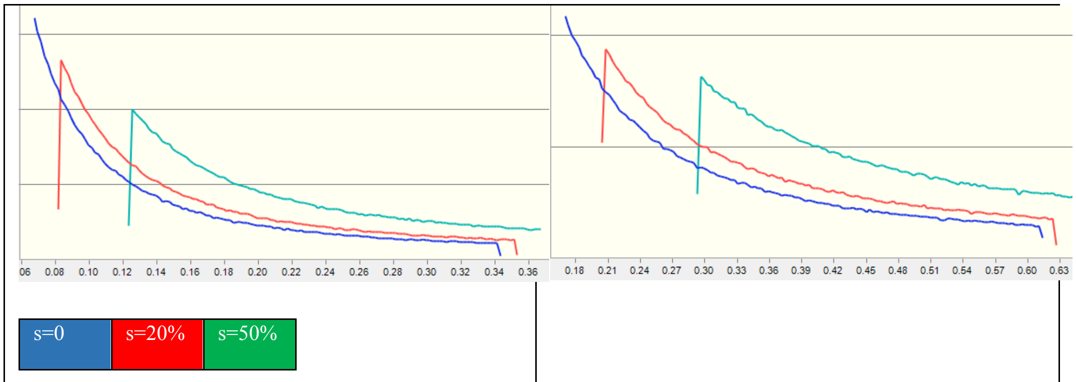

Figure 13. Sensitivity to interest rate for wage growth. The Y axis shows the interest rate varying from 1% to 20%. X axis on the left shows growth at the 10th period and on the right growth at 20th period.

4. **Wage growth sensitivity to product elasticity of human capital sensitivity**: in the simulations presented in Figures 6, we used *γ=0.3.* Figure 14 shows growth as a function of γ. The Y axis shows γ varying from 0.1 to 0.3 and the X axis shows growth at the 10th and 20th periods for three different subsidy rate: 0, 20% and 50%. We see that growth in all cases is highest for a subsidy rate of 50%.

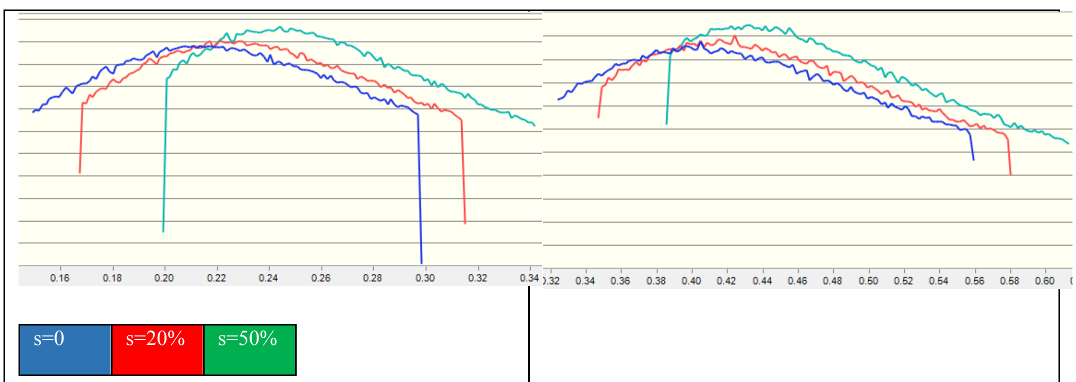

Figure 14. Sensitivity to product elasticity of human capital for wage growth. The Y axis shows gamma varying from 0.1 to 0.3. X axis on the left shows growth at the 10th period and on the right growth at 20th period.

5. **Income per capita sensitivity to the interest rate**: in the simulations presented in Figures 7, we used *r=2%.* Figure 15 shows growth as a function of interest rate. The Y axis shows interest rate varying from 1% to 20% and the Y axis shows I/C

at the 10th and 20th periods for three different subsidy rate: 0, 20% and 50%. We see that when interest rate drops I/C increases in all cases and stay slightly higher for a subsidy rate of 50%.

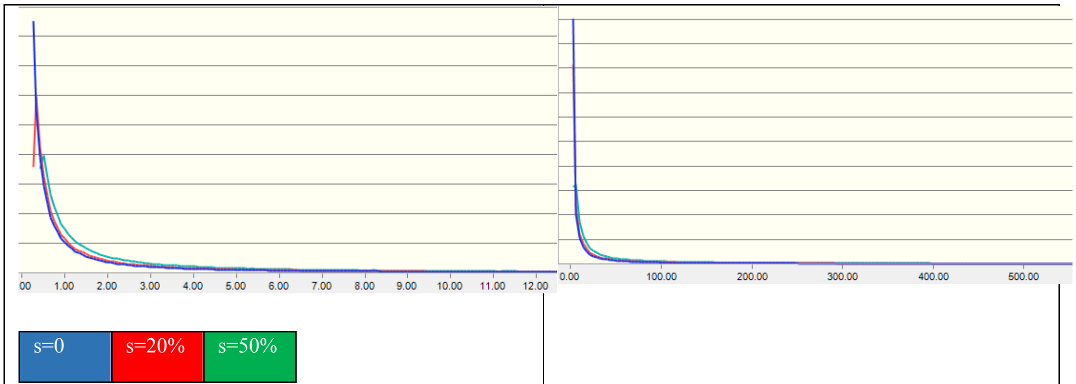

Figure 15. Sensitivity to interest rate for income per capita. The Y axis shows the interest rate varying from 1% to 20%. X axis on the left shows growth at the 10th period and on the right growth at 20th period.

6. **Income per capita sensitivity to product elasticity of human capital sensitivity**: in the simulations presented in Figures 7, we used *γ=0.3.* Figure 16 shows I/C as a function of γ. The Y axis shows γ varying from 0.1 to 0.3 and the X axis shows growth at the 10th and 20th periods for three different subsidy rate: 0, 20% and 50%. We see that I/C in all cases is slightly higher for a subsidy rate of 50%.

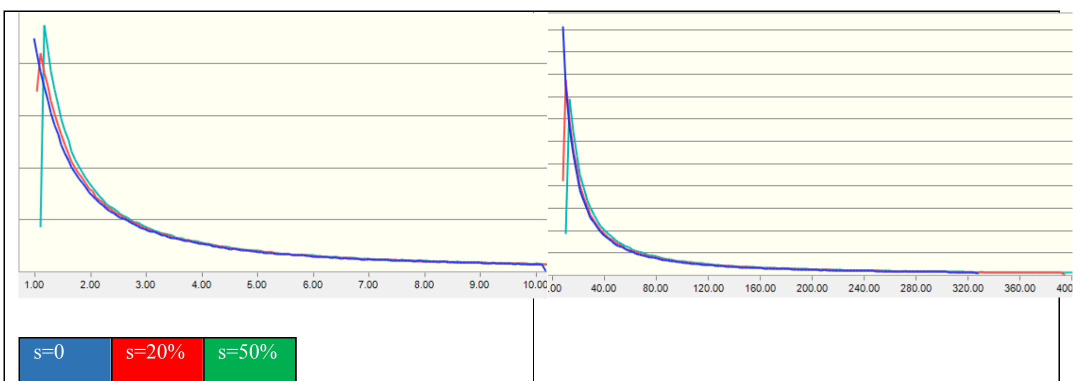

Figure 16. Sensitivity to product elasticity of human capital for income per capita. The Y axis shows gamma varying from 0.1 to 0.3. X axis on the left shows growth at the 10th period and on the right growth at 20th period.

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